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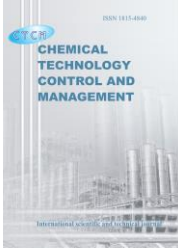

Since 2005

REGULAR ALGORITHMS FOR ADAPTIVE IDENTIFICATION OF LINEAR NON-STATIONARY SYSTEMS

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Abstract: *The article deals with the formation of regular algorithms for adaptive parametric spline identification of linear non-stationary systems. Analyzed the issues of constructing adaptive identification of continuous linear non-stationary systems with monotonic and sign-constant parameters. In this case, in a non-stationary system, the entire observation interval is in some way divided into sub-intervals. Identification is carried out at each of the parameter constancy intervals, while the parameter matrix is re-evaluated for the next interval. The input data of the system are determined with an error depending on the error of approximating the state of the system by splines and approximating the ratios by numerical integration formulas. To solve systems of linear algebraic equations, projection algorithm was used, which makes it possible to facilitate the control of the accuracy of intermediate calculations. The above computational procedures make it possible to regularize the problem of adaptive parametric spline identification of linear non-stationary systems based on an iterative algorithm and improve the quality indicators of control processes.*

Keywords: *linear nonstationary system, adaptive parametric spline identification, stable algorithms, iterative algorithm, regularization.*

Аннотация: *Мақолада чизиқли ностационар тизимларни адаптив параметрик сплайн идентификациялашнинг мунтазам алгоритмларини шакллантириш масалалари кўриб чиқилган. Монотон ва ишораси ўзгармас параметрли узлуксиз чизиқли ностационар тизимларнинг адаптив идентификациясини қуриш масалалари таҳлил қилинган. Бундай ҳолда, ностационар тизимда бутун кузатиш оралиғи қайсидир маънода кичик интервалларга бўлинади. Идентификациялаш ҳар бир параметр доимийлиги оралиғида амалга оширилади, параметр матрицаси эса кейинги интервал учун қайта баҳоланади. Тизимнинг кириш маълумотлари тизим ҳолатини сплайнлар орқали яқинлаштириш ва сонли интеграция формулалари билан нисбатларни яқинлашиш хатосига қараб хатолик билан аниқланади. Чизиқли алгебраик тенгламалар тизимини ечиш учун проекцияли алгоритмлардан фойдаланилган, бу эса оралиқ ҳисоб-китобларнинг аниқлигини назорат қилишни осонлаштириш имконини беради. Юқоридаги ҳисоблаш амаллари итератив алгоритмга асосланган чизиқли ностационар тизимларни адаптив параметрик сплайн идентификациялаш муаммосини тартибга солиш ва назорат қилиш жараёнларининг сифат кўрсаткичларини яхшилаш имконини беради.*

Таянч сўзлар: *чизиқли ностационар тизим, адаптив параметрик сплайн идентификациялаш, турғун алгоритмлар, итерацияли алгоритм, мунтазамлаштириш.*

Аннотация: *В статье рассматриваются вопросы формирования регулярных алгоритмов адаптивной параметрической сплайн-идентификации линейных нестационарных систем. Анализируются вопросы построения адаптивной идентификации непрерывных линейных нестационарных систем с монотонными и знакопостоянными параметрами. В этом случае в нестационарной системе весь интервал наблюдения некоторым образом разбивается на подинтервалы. Идентификация осуществляется на каждом из интервалов постоянства параметров, при этом для последующего интервала заново оценивается матрица параметров. Входные данные системы определяются с погрешностью, зависящей от погрешности приближения состояния системы сплайнами и аппроксимации соотношений формулами численного интегрирования. Для решения систем линейных алгебраических уравнений использован проекционный алгоритм, позволяющий облегчить контроль точности промежуточных вычислений. Приведенные вычислительные процедуры позволяют регуляризировать задачу адаптивной параметрической сплайн-идентификации линейных нестационарных систем на основе итерационного алгоритма и повысить качественные показатели процессов управления.*

Ключевые слова: *линейная нестационарная система, адаптивная параметрическая сплайн-идентификация, устойчивые алгоритмы, итерационный алгоритм, регуляризация.*

Introduction

The problems of system synthesis that arise in engineering practice can be grouped into two classes, classifying general problems in the first class and special ones in the second [1]. For general synthesis problems, there is a characteristic lack of connection with the specifics of the functioning of the system - with its purpose. In special synthesis problems, either it is required to determine the mathematical model of the system that satisfies the requirements for the processes, or to optimize the mathematical model, i.e., to choose some of the best from a given set of models (the problem of the optimal system). Both the requirements for processes and the specific meaning of the concept of optimality are related to the purpose of the system. They are different, for example, for an automatic control system and for an information system, for various types of automatic control systems: stabilization systems, program control systems, tracking systems, extreme control systems, end state control systems [1-3].

If signals (or characteristics) are accepted as input and output signals (or their characteristics) that have identical mathematical descriptions with the corresponding signals (or their characteristics) at the inputs and outputs of a real system, and the latter are established from the experiment, then this is the problem of system identification [2].

At present, a large number of different formulations of the identification problem are known. One of the distinguishing features of these formulations is the proposed class of systems in which the model of the identified system is sought [4].

Formulation of the problem

Let us represent the mathematical model of a non-stationary system in the following form:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t),\ t \in \left[t_0, T_f\right],\ x(t_0) = x^{(0)}, \tag{1}$$

there in $A(t) = \{a_{ij}(t)\},\ B(t) = \{b_{ik}(k)\}$ – two matrices that are sized respectively $n \times n$ and $n \times m$, matrix data elements are of constant sign

$$sign\left[a_{ij}(t)\right] = const,\quad sign\left[b_{ik}(t)\right] = const, \tag{2}$$

and monotonic functions

$$sign\left[da_{ij}(t)/dt\right] = const,\quad sign\left[db_{ik}(t)/dt\right] = const. \tag{3}$$

The first derivatives of the data elements of the matrices are continuous, and they are also defined in limited areas in the time interval $\left[t_0, T_f\right]$.

The systems described by (1)-(3) are quasi-stationary [4-7]. Let us divide in some way the entire observation interval $\left[t_0, T_f\right]$ into subintervals $\left[t_l, t_l + T\right]$. In these subintervals, the coefficients of the matrices $A(t),\ B(t)$ are $a_{ij}(T_l),\ b_{ik}(T_l)\ (i, j = \overline{1,n}),\ (k = \overline{1,m})$ respectively, can be considered immutable. Identification occurs at each of the intervals where the parameters remain constant. For each of the following intervals, the estimation of the parameter matrix will occur again, while those data that do not belong to the considered subinterval will be completely ignored [8]. On separate subintervals, the coefficients $a_{ij}(T_l),\ b_{ik}(T_l)\ (l = \overline{1,L})$ can be different.

If the control vector $u(t)$ is given, respectively, the state vector function $x(t)$ on the segment $\left[t_l, t_l + T\right]$ is defined by its eigenvalues $x^{(i)}x(t_i),\ t_i \in \left[t_l, t_l + T\right]\ (i = \overline{0,N})$, then for the system (1)-(3) under consideration, such an identification problem will consist in determining estimates $\hat{a}_{ij}^l = \hat{a}_{ij}(T_l),\ \hat{b}_{ik}^l = \hat{b}_{ik}(T_l)\ (i, j = \overline{1,n}),\ (k = \overline{1,m})$ for the parameters $a_{ij}(T_l),\ b_{ik}(T_l)$ of the matrices A and B, which are unknown [5,8,9]. These parameters provide the minimum of the quadratic criterion (5), which is considered within the subinterval $\left[t_l, t_l + T_l\right]$.

In general, model (1)-(3) will have identifiable parameters $n\times(n+m)$ These parameters can be described by a matrix that will have the size $(n+m)\times n$

$$G(t)=\left[g^{(1)}\dots g^{(i)}\dots g^{(n)}\right], \tag{4}$$

where $g^{T(i)}=\{a_{i1}(t),\dots,a_{ij}(t),\dots,a(t),b_{i1}(t),\dots,b_{ik}(t),\dots,b_{\wp}(t)\}-(n+m)-$dimensional vector that corresponds to the i -th rows of matrices $A(t)$, $B(t)$.

Therefore, the problem of identifying linear non-stationary systems (1)-(3) is determined by finding an estimate $G(t)$ of the parameter matrix (4), which provides a minimum of the residual criterion

$$Q(\hat{G})\min_{G}\ Q(G)=\min_{G}\left\{\int_{t_0}^{T_f}\left[\dot{\bar{x}}(t)-A(t)\bar{x}(t)-B(t)\bar{u}(t)\right]^2 dt\right\}. \tag{5}$$

We will use cubic splines [7-11]. With their help, we interpolate the values of the vector $\bar{x}^{(i)}\ (i=0,N)$

$$\hat{x}_1(t)\to S_1(t),\dots,\hat{x}_i(t)\to S_i(t),\dots,\hat{x}_n(t)\to S_n(t).$$

We obtain an analytical expression $\hat{\bar{x}}$ that evaluates the state vector function, i.e., we pass from $\bar{x}(t_i)\ (i=0,N)$ to $\bar{S}(t)$, where is $\bar{S}(t)$, an n-dimensional vector function, and each component $\bar{S}_i(t)\ \ (i=1,N)$ will be a cubic spline function. The approximation $\dot{x}_i(t)-\hat{\dot{x}}(t)$ is defined as a derivative of a spline $S_i(t)-S_l(t)$ [8, 11]. Now the mathematical model for the non-stationary system (1) - (3) is transformed to the form:

$$\dot{S}_i(t)=\sum_{j=1}^{n}a_{ij}^{l}S_j(t)+\sum_{k=1}^{m}b_{ik}^{l}u_k(t)\quad (i=\overline{1,n}),\ t\in\left[t_l,t_l+T_l\right]. \tag{6}$$

For the integral quadratic residual criterion (5) of model (1) - (3), the equation will take the form:

$$Q_i(g_l^i)=\int_{t_l}^{t_l+T_l}\left[\dot{S}_i(t)-\sum_{j=1}^{n}a_{ij}^{l}S_j(t)-\sum_{k=1}^{m}b_{ik}^{l}u_k(t)\right]^2 dt\qquad (i=\overline{1,n}). \tag{7}$$

We differentiate the discrepancies $Q_i\ \ (i=\overline{1,N})$ by the elements of the matrices A and B, while using the necessary conditions for the minimum of the functional (7) [7-11], and equating to zero

$$\frac{\partial Q_i(g_l^{(i)})}{\partial a_{ij}^{l}}=0,\qquad \frac{\partial Q_i(g_l^{(i)})}{\partial b_{ik}^{l}}=0,\qquad (j=\overline{1,n}),\ \ (k=\overline{1,m})$$

For the minimum of such a discrepancy Q_i

$$\int_{t_l}^{t_l+T_l}\left[\dot{S}(t)-\sum_{j=1}^{n}\hat{a}_{ij}^{l}S_j(t)-\sum_{k=1}^{m}\hat{b}_{ik}^{l}u_k(t)-S_j(t)\right]^2 dt=0,$$

$$\int_{t_l}^{t_l+T_l}\left[\dot{S}_i(t)-\sum_{j=1}^{n}\hat{a}_{ij}^{l}S_j(t)-\sum_{k=1}^{m}\hat{b}_{ik}^{l}u_k(t)-u_k(t)\right]^2 dt=0,$$

$$(j=\overline{1,n}),\ \ (k=\overline{1,m}).$$

Let's carry out a number of transformations in order to determine the vector of estimates $g_l^{(i)}$ on the subinterval $\left[t_l,t_l+T_l\right]$. As a result, we obtain a system of linear algebraic equations

$$C^{(i)}\bar{g}_l^{(i)}=d^{(i)}, \tag{8}$$

here $C^{(i)}$ is a matrix of size $(n+m)\times(n+m)$, which has a block structure:

$$C^{(i)}=\begin{bmatrix} C_{(n\times n)}^{(i)} & \vdots & C_{(n\times m)}^{(i)} \\ \cdots & \cdots & \cdots \\ C_{(m\times n)}^{(i)} & \vdots & C_{(m\times m)}^{(i)} \end{bmatrix}.$$

The elements of this matrix can be defined as:

$$C^{i}_{(n\times n)} = \left\{c^{(i)}_{i_1,j}\right\},\ c^{(i)}_{i_1,j} = \int_{t_l}^{t_l+T_l} S_{i_1}(t)\,S_j(t)dt, \tag{9}$$

$$C^{i}_{(n\times m)} = \left\{c^{(i)}_{i_1,k}\right\},\ c^{(i)}_{i_1,k} = \int_{t_l}^{t_l+T_l} S_{i_1}(t)\,u_k(t)dt, \tag{10}$$

$$C^{i}_{(m\times n)} = \left\{c^{(i)}_{kj}\right\},\ c^{(i)}_{kj} = \int_{t_l}^{t_l+T_l} u_k(t)S_j(t)dt, \tag{11}$$

$$C^{i}_{(m\times m)} = \left\{c^{(i)}_{i_2,k}\right\},\ c^{(i)}_{i_2,k} = \int_{t_l}^{t_l+T_l} u_{i_2}(t)u_k(t)dt; \tag{12}$$

where $\bar{d}_l^{(i)T} = \left\{d_1^{(i)},\ldots,d_n^{(i)},d_{n+1}^{(i)},\ldots,d_{n+m}^{(i)}\right\} - (n+m)$ - dimensional vector of free terms.

We define the elements of this vector as

$$d^{(i)}_{l_1} = \int_{t_l}^{t_l+T_l} S_{l_1}(t)\,\dot{S}_i(t)dt,\ \ (l_1 = \overline{1,n}), \tag{13}$$

$$d^{(i)}_{l_2} = \int_{t_l}^{t_l+T_l} u_{l_2-n}(t)\dot{S}_i(t)\,dt\ \ (l_2 = \overline{n+1,\ n+m}), \tag{14}$$

where $\bar{g}_l^{(i)} = \left\{a_{il}^l,\ldots,a^l,b_{il}^l,\ldots,b_{n+m}^l\right\} - (n+m)$ -dimension vector defined in the interval of quasi-stationarity $\left[t_l, t_l + T_l\right]$ and used to estimate unknown parameters.

The input data are the coefficients of the matrix and the right side of the system of linear algebraic equations (8), determined with a certain error. Moreover, this error depends on the approximation of the state of the system (1) - (3) by splines, and, in addition, on the replacement of relations (9) - (14), produced by numerical integration formulas.

As a result, the algorithm for estimating the parameters of system (6), which is linear and quasi-stationary, was transformed to the solution of a system of n linear algebraic equations (8) for each interval of parameter constancy T_l.

To divide the observation interval $\left[t_0 + T_f\right]$, two methods are usually used [7, 8, 11]:

- algorithm that uses a fixed partition of the interval. This method will be effective if there is some a priori information showing the dynamics of changes in parameters that are unknown over time. In this case, the observation interval has a good uniform or non-uniform division. To choose the duration of the quasi-stationarity interval, it is assumed that $T_l \geq t_{рег}$, here $t_{рег}$ is the time of the transient process;
- algorithm using adaptive partition selection. Using the current information about the behavior of the system, that is, observing the state at some points, one can adaptively choose the value of time intervals using a piecewise constant approximation of unknown parameters.

Solution

When solving equation (8), the inverse matrix is calculated, which causes computational difficulties, since the size of the $C^{(i)}$ matrix is large and it can be degenerate. Below we consider an algorithm for solving systems of equations of the form (8), based on the use of the Kachmac's projection algorithm [12-15]. The use of this algorithm is practically convenient and suitable for a wide range of ratios between the required and available computer RAM, which makes it easier to control the accuracy of intermediate calculations and, if necessary, to regularize the calculated solution.

The sequence of approximations $\bar{g}^{(i)}_{l,k}$ calculated by the formula

$$\bar{g}^{(i)}_{l,k} = \bar{g}^{(i)}_{l,k-1} + \left[C^{(i)}_\nu\right]^{+}(d^{(i)}_\nu - C^{(i)}_\nu \bar{g}^{(i)}_{l,k-1}),\ \ \nu = \mathrm{mod}_p(k), \tag{15}$$

converges in the Euclidean norm to the solution $\overline{g}_{l,*}^{(i)}$ of system (8). The $C^{(i)}$ matrix and $d^{(i)}$ vector are represented as follows:

$$C^{(i)} = \begin{bmatrix} C_1^{(i)} \\ \dots \\ C_p^{(i)} \end{bmatrix},\ d^{(i)} = \begin{bmatrix} d_1^{(i)} \\ \dots \\ d_p^{(i)} \end{bmatrix},$$

where $\overline{g}_{l,0}^{(i)}$ – arbitrary vector.

Consider the question of the dependence of the rate of convergence of the algorithm on the slopes of linear manifolds [12, 16-19]. Let us introduce the notation $r_{k,k-1} = \overline{g}_{l,k-1}^{(i)} - \overline{g}_{l,k}^{(i)}, r_k = \overline{g}_{l,k}^{(i)} - \overline{g}_{l,*}^{(i)}$. From (15) it follows

$$r_{k,k-1} = -\left[C_\nu^{(i)}\right]^+ (d_\nu^{(i)} - C_\nu^{(i)}\overline{g}_{l,k-1}^{(i)}) = \left[C_\nu^{(i)}\right]^+ C_\nu^{(i)}\overline{g}_{l,k-1}^{(i)} - \left[C_\nu^{(i)}\right]^+ d_\nu^{(i)}. \quad (16)$$

Known [20,21], that $\left[C_\nu^{(i)}\right]^+ d_\nu^{(i)} \in R_\nu$, $\left[C_\nu^{(i)}\right]^+ C_\nu^{(i)}\overline{g}_{l,k-1}^{(i)} \in R_\nu$ therefore, $r_{k,k-1} \in R_\nu$. Subtracting the vector from the right and left parts of equality (15) and taking into account $C_\nu^{(i)}\overline{g}_{l,*}^{(i)} = d_\nu^{(i)}$ that we get

$$r^i = \overline{g}_{l,k}^{(i)} - \overline{g}_{l,*}^{(i)} = \overline{g}_{l,k-1}^{(i)} - \overline{g}_{l,*}^{(i)} + \left[C_\nu^{(i)}\right]^+ d_\nu^{(i)} - \left[C_\nu^{(i)}\right]^+ C_\nu^{(i)}\overline{g}_{l,k-1}^{(i)} = (I - \left[C_\nu^{(i)}\right]^+ C_\nu^{(i)})\overline{g}_{l,k-1}^{(i)} - (I - \left[C_\nu^{(i)}\right]^+ C_\nu^{(i)})\overline{g}_{l,*}^{(i)}.$$

Subtracting the vector $\overline{g}_{l,*}^{(i)}$ from both parts of (15) and taking into account (16), we obtain

$$r_k = r_{k-1} - r_{k,k-1}, \quad (17)$$

$$r_{k,k-1} = \left[C_\nu^{(i)}\right]^+ C_\nu^{(i)} r_{k-1}.$$

Using the known properties of the norm [20, 21], we obtain

$$\frac{\left\|r_{k,k-1}\right\|}{\left\|r_{k-1}\right\|} \geq \theta(N(C_\nu^{(i)}), N(C_{\nu-1}^{(i)})),$$

where $N(C_\nu^{(i)})$, $N(C_{\nu-1}^{(i)})$ – are operator kernels $C_\nu^{(i)}$ and $C_{\nu-1}^{(i)}$, $\theta = \min_{s \in N_l} \left\|(I - \left[C_\nu^{(i)}\right]^+ C_\nu^{(i)})\overline{g}_l^{(i)}\right\|$.

The calculation process according to formula (15) should be performed until the error satisfies the condition

$$\left\|r_k\right\| = \left\|\overline{g}_{l,k}^{(i)} - \overline{g}_{l,*}^{(i)}\right\| \leq \delta.$$

But the exact value $\overline{g}_{l,*}^{(i)}$ is not known, so another stopping criterion is usually used - the fulfillment of the inequality

$$\left\|r_{k,k-1}\right\| = \left\|\overline{g}_{l,k-1}^{(i)} - \overline{g}_{l,k}^{(i)}\right\| \leq \varepsilon. \quad (18)$$

The vectors r_k and r_k are related by relation (17), so it is possible to estimate the error of the obtained approximate solution using expression (18).

Conclusion

The above computational procedures make it possible to regularize the problem of adaptive parametric spline identification of linear non-stationary systems based on an iterative algorithm and improve the quality indicators of control processes.

References:

1. N.D.Egupov, K.A.Pupkov, *Methods of classical and modern theory of automatic control*, Textbook in 5 volumes. Moscow: Publishing house of MSTU named after N.E. Bauman, 2004.
2. V.D.Yurkevich, *Synthesis of non-linear non-stationary control systems with multi-tempo processes*. St. Petersburg: Nauka, 2000.
3. I.V.Miroshnik, V.O.Nikiforov, A.L.Fradkov, *Nonlinear and adaptive control of complex dynamic systems*, St.Petersburg: Nauka, 2000, 549 p.

4. E.G.Kleiman, “Identification of non-stationary objects”, *Avtomat. i telemekh.*, issue 10, pp. 3–45, 1999.
5. N.N.Karabutov, *Adaptive system identification: information synthesis*. 2006, 384 p.
6. V.M.Kuntsevich, *Control under Uncertainty: Guaranteed Results in Control and Identification Problems.* Kiev: Naukova Dumka, 2006, 264 p.
7. N.V.Plotnikova, N.S.Kalistratova, O.N.Malyavkin, “Algorithms for solving the identification problem”, *Computer technologies*, *management, radio electronics*. issue 4, Bulletin of SUSU, no. 14, pp. 133-139, 2006.
8. A.A.Stenin, M.M.Tkach, E.Yu.Melkumyan, “Generalized identification algorithm for linear dynamic systems based on spline functions and Walsh functions”, *ASAU.* no. 20(40), pp. 131-135, 2012.
9. I.G.Burova, Yu.K.Demyanovich, *Minimal splines and their applications*. St. Petersburg: Publishing House of St. Petersburg. un-ta, 2010, 364 p.
10. I.G.Burova, *Approximation by real and complex minimal splines*. St. Petersburg: Ed. house of the St. Petersburg state. un-ta, 2013, 141 p.
11. L.I.Konstantinova, V.A.Kochegurov, B.M.Shumilov, “Parametric identification of nonlinear differential equations based on spline schemes exact on polynomials”, *Avtomat. i telemekh.*, issue 5, pp. 53-63, 1997.
12. S.Kaczmarz, “Approximate solution of systems of linear equations”, *Internat. J. Control*. Vol. 57, no. 6, pp. 1269-1271, 1993.
13. V.P.Ilyin, “On the iteration method of Kaczmarz and its generalizations”, *Sib. magazine industry Mat.*, Vol. 9(3), pp. 39-49, 2006.
14. J.Saad, *Iterative Methods for Spars Linear Systems*. Boston: EPS, 1996.
15. U.F.Mamirov, *Regular synthesis of adaptive control systems for uncertain dynamic objects*. Tashkent: “Bilim va intellektual salohiyat”, 2021, 215 p.
16. V.N.Babenko, “Convergence of Kaczmarz's projection algorithm”, *Zh. Vychisl. math. and mat. fiz.*, Vol. 24(10), pp. 1571-1573, 1984.
17. H.Z.Igamberdiyev, U.F.Mamirov, “Sustainable estimation of parameters and covariation of disturbance vectors in uncertain systems”, *Chemical Technology, Control and Management*. Vol. 2018: Iss. 3, Article 4, pp. 16-19, 2018. DOI: https://doi.org/10.34920/2018.4-5.16-19.
18. N.Yusupbekov, H.Igamberdiev, U.Mamirov, “Algorithms for Robust Stabilization of a Linear Uncertain Dynamic Object Based on an Iterative Algorithm”, *In: Kahraman C., Cebi S., Cevik Onar S., Oztaysi B., Tolga A.C., Sari I.U. (eds) Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021. Lecture Notes in Networks and Systems*, vol 307. Springer, Cham. https://doi.org/10.1007/978-3-030-85626-7_28
19. N.R.Yusupbekov, H.Z.Igamberdiev, U.F.Mamirov, “Stable Algorithms for Adaptive Control and Adaptation of Uncertain Dynamic Objects Based on Reference Models”, *CEUR Workshop Proceedings*, Vol. 2965, Kolomna, Russia, pp. 296-302, 2021.
20. Ch.Lawson, R.Henson, *Numerical solution of problems in the method of least squares*, 1986, 232 p.
21. F.R.Gantmacher, J.L.Brenner, *Applications of the Theory of Matrices*, Courier Corporation, 2005.