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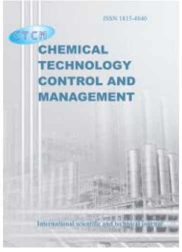

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ALGORITHMS FOR STRUCTURAL SYNTHESIS OF CONTROL SYSTEMS FOR DYNAMIC OBJECTS UNDER CONDITIONS OF UNMEASURABLE DISTURBANCES

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Abstract: *The paper talks about ways to use inverse dynamic models to structure a control system for moving objects when disturbances are not measured. To solve the main problems that arise when implementing the inverse dynamic model method, methods for synthesizing inverse systems with given dynamic characteristics are used. The solution to this problem is obtained based on the theory that dynamic observers are invariant to unmeasured input influences. We broke down the matrix operator on the right into elementary orthogonal rotation matrices and on the left into permutation matrices to get a stable answer to the equation of the current value of an unmeasured input signal. These given algorithms can be used to solve the structural synthesis of an inverse system as long as the inputs to the original system can be reversed or observed and there are no more unmeasured variables in the inputs than there are unmeasured variables in the outputs.*
Key words: *dynamic object, control system, structural synthesis algorithms, unmeasured disturbances, dynamic observer, inverse dynamic model method, pseudo-inverse matrix.*

Annotatsiya: *Teskari dinamik modellar asosida o'lchanmaydigan g'alayonlar sharoitida dinamik ob'yektlarni boshqarish tizimining strukturaviy sintezlash algoritmlari keltirilgan. Teskari dinamik modellar usulini amalga oshirishda yuzaga keladigan asosiy muammolarni hal qilish uchun oldindan berilgan dinamik xususiyatlarga ega teskari tizimlarni sintezlash usullari qo'llaniladi. Masalaning echimi o'lchanmaydigan kirish ta'siriga invariant bo'lgan dinamik kuzatuvchilar nazariyasi asosida olingan. O'lchanmaydigan kirish signalining joriy qiymati tenglamasining turg'un yechimini olish uchun matritsa operatorining o'ng tomonda elementar ortogonal aylantirish matritsalariga va chap tomonda o'zgarish matritsalariga bo'linishi qo'llanilgan. Teskari tizimning strukturaviy sintezlash uchun keltirilgan algoritmlar dastlabki tizimning kirish qismida qaytarilish yoki kuzatish shartlari bajarilganda, ya'ni o'lchanmaydigan kirish o'zgaruvchilari soni chiqish o'zgaruvchilari sonidan oshmalik shartlarida yeccchilishi mumkin.*
Tayanch so'zlar: *dinamik ob'yekt, boshqarish tizimi, strukturaviy sintez, o'lchanmadigan g'alayonlar, dinamik kuzatuvchi, teskari dinamik model usuli, mavhum teskari matritsa.*

Аннотация: *Приведены алгоритмы структурного синтеза системы управления динамическими объектами в условиях неизмеряемых возмущений на основе обратных динамических моделей. Для решения сформулированных основных проблем, возникающих при реализации метода обратных динамических моделей, используются методы синтеза обратных систем с заданными динамическими характеристиками. Решение задачи получено на основе теории динамических наблюдателей, инвариантных к неизмеряемым входным воздействиям. Для получения устойчивого решения уравнения текущего значения неизмеряемого входного сигнала использовано разложение матричного оператора справа на элементарные ортогональные матрицы вращения и слева на матрицы перестановок. Приведенные алгоритмы структурного синтеза обратной системы разрешимы при выполнении условий обратимости или наблюдаемости по входу исходной системы, т.е. когда число входных неизмеряемых переменных не превышает числа выходных.*

Ключевые слова: *динамический объект, система управления, структурный синтез, неизмеряемые возмущения, динамический наблюдатель, метод обратной динамической модели, псевдообратная матрица.*

1. Introduction:

Inverse dynamic models are very important for figuring out how to reconstruct input signals from indirect measurements. This is what makes it possible to account for disturbances that cannot be measured [1-3]. However, the method cannot be used very often because the equations of the control object determine the structure and parameters of inverse models, as well as how they move.

We will call invariant dynamic observers that provide an estimate of the state vector of an object for which the estimation error does not depend on the unmeasured input signal. In this case, it becomes possible, in principle, to evaluate the specified signal using the object's equation and estimates of its state vector. The corresponding dynamic equations of the invariant observer and the static equations for estimating the input signal together form the equations of the synthesized inverse system. If the specified synthesis problem is solvable, then it has more than one solution, which makes it possible to parameterize the inverse system equation [3-5]. Thus, the stage of structural synthesis is reduced to obtaining the equations of the inverse system accurate to a set of varied parameters. In the parametric synthesis stage, the chosen parameters are chosen based on the conditions for making sure the reverse system has the desired dynamic properties. These include requirements for the quality of transient processes and requirements for noise immunity when measurement errors happen.

Let us obtain a solution to the problem of the structural synthesis of an inverse system as applied to a discrete dynamic object. Consider a discrete dynamic system of the form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + B_0 u_k^0, \\ y_k &= Cx_k + Du_k + D_0 u_k^0, \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where $x_k \in R^n$ – the state vector, $u_k \in R^m$ – vector of unmeasured input signals, $u^0 \in R^{m_0}$ – vector of measured input signals, $y_k \in R^q$ – vector of measured output signals.

2. Structural synthesis of an inverse system of complete order.

Let us construct a dynamic observer of the state vector of the system (1), invariant to the unmeasured input signal. Let us take the observer equation in the form

$$\begin{aligned} \tilde{x}_{k+1} &= F\tilde{x}_k + Gy_k + G_0 u_k^0, \\ \hat{x}_k &= \tilde{x}_k + Hy_k + H_0 u_k^0, \\ \tilde{x}(0) &= \tilde{x}_0, \end{aligned} \tag{2}$$

where $\tilde{x}_k \in R^n$ – observer state vector, F, G, G_0, H, H_0 – matrices of parameters of corresponding dimensions to be determined.

Equations (2) can be represented in the equivalent form

$$\hat{x}_{k+1} = F\hat{x}_k + (G - FH)y_k + Hy_{k+1} + (G_0 - FH_0)u_k^0 + H_0 u_{k+1}^0. \tag{3}$$

Let us select the matrices with different parameters so that the estimation error of the state vector $e_k^x = x_k - \hat{x}_k$ is independent of u_k. Then, to determine the current value of the input signal u_k, you can use the system of linear algebraic equations:

$$\begin{pmatrix} B \\ \hdashline D \end{pmatrix} u_k = \begin{pmatrix} \hat{x}_{k+1} - A\hat{x}_k - Bu_k^0 \\ \hdashline y_k - C\hat{x}_k - D_0 u_k^0 \end{pmatrix} \tag{4}$$

3. Problem definition

System (4) is solvable with respect to $u(k)$ if the condition is satisfied $rank\left(B^T \;\vdots\; D^T\right) = m$ and its normal pseudosolution can be taken as an estimate for $\hat{u}_k$. Taking into account (3) and the known formulas for pseudo-inversion of block matrices [6-8]:

$$\begin{pmatrix} B \\ \hdashline D \end{pmatrix}^+ = \left(\tilde{B} \;\vdots\; \tilde{D}\right), \;\; \tilde{B} = P_D\left(BP_D\right)^+, \;\; P_D = I_m - D^+D, \;\; \tilde{D} = D^+ - P_D\left(BP_D\right)^+ BD^+, \qquad (5)$$

to obtain an expression for the estimate

$$\begin{aligned} \tilde{u}_k &= \left[\tilde{B}(F - A) - \tilde{D}C\right]\hat{x}_k + \left[\tilde{B}(G - FH) + \tilde{D}\right]y_k + \\ &+ \tilde{B}Hy_{k+1} + \left[\tilde{B}(G_0 - FH_0 - B_0) - \tilde{D}D_0\right]u_k^0 + \tilde{B}H_0u_{k+1}^0. \end{aligned} \qquad (6)$$

From (1) and (3), it follows that the error in estimating the state vector satisfies the difference equation:

$$\begin{aligned} e_{k+1}^x &= Fe_k^x + \left[(I_n - HC)A - F(I_n - HC) - GC\right]x_k + \\ &+ \left[(I_n - HC)B - (G - FH)D\right]u_k + \\ &+ \left[(I_n - HC)B_0 - (G - FH)D_0 - (G_0 - FH_0)\right]u_k^0 - \\ &- (HD_0 + H_0)u_{k+1}^0, \end{aligned} \qquad (7)$$

where $e_0^x = x_0 - \hat{x}_0$.

From (7), we obtain the conditions for the invariance of the estimation error in the form

$$\begin{aligned} &(I_n - HC)A - F(I_n - HC) = GC, \\ &(I_n - HC)B - (G - FH)D = 0, \\ &(I_n - HC)B_0 - (G - FH)D_0 - G_0 + FH_0 = 0, \\ &HD_0 + H_0 = 0, \;\; HD = 0, \end{aligned} \qquad (8)$$

which is a generalization of the well-known Luenberger conditions for the case of unmeasured influences [9, 10].

If conditions (8) are met, equations (3), and (5) can be interpreted as equations of an inverse system, the task of structural synthesis of which is reduced to finding the corresponding parameter matrices by solving a system of linear matrix equations (8). In this case, the conditions for the solvability of the specified system and the type of solution depend on the relationship between the structural parameters of the system (1).

Here system (8) is solvable if $rank\, D = m$. Then the most general solution (8) has the form:

$$\begin{aligned} &F = A - BD^+C - L\Omega_DC, \quad G = BD^+ + L\Omega_D, \\ &H = H_0 = 0, \quad \Omega_D = I_q - DD^+, \\ &G_0 = B_0 - BD^+D_0 - L\Omega_DD_0. \end{aligned} \qquad (9)$$

where $L \in R^{n\times q}$ – arbitrary matrix of variable parameters. Obviously, since in this case $P_D = 0$, then $\tilde{B} = 0$, $\tilde{D} = D^+$.

As a result, the equation of the inverse system (3), (6) is transformed to the form

$$\begin{aligned} x_{k+1}^I &= \left(A - BD^+C\right)x_k^I + L\Omega_D\left[y_k - Cx_k^I\right] + \\ &+ BD^+y_k + \left(BD^+D_0 + L\Omega_DD_0 - B_0\right)u_k^0, \\ u_k^I &= -D^+Cx_k^I + D^+y_k - D^+D_0u_k^0, \end{aligned} \qquad (10)$$

where $x_I(k) = \hat{x}(k), \quad u_I(k) = \hat{u}(k)$.

If $D = 0$, then the condition for the solvability of system (8) is $rank(CB) = m$. Then the corresponding solution can be obtained in the following form:

$$
\begin{aligned}
&F = (I_n - L'C)\Pi A - L''C, \qquad H = B(CB)^+ + L'\Omega,\\
&G = (I_n - L'C)\Pi AH - L''(I - CL')\Omega,\\
&\Pi = I_n - B(CB)^+ C, \qquad \Omega = I_q - (CB)(CB)^+,\\
&G_0 = (I_n - L'C)\Pi B_0 - GD_0, \qquad H_0 = -HD_0,
\end{aligned} \tag{11}
$$

where $L', L'' \in R^{n\times q}$ – arbitrary matrices. Because $C\Pi = \Omega C$, then $F = \Pi A - LC,\ L = L'\Omega A + L''$, and without loss of generality, we can put $L' = 0$.

Taking into account the fact that in the case under consideration $\tilde{B} = B^+,\ \tilde{D} = 0$, the equations of the inverse system will take the form

$$
\begin{aligned}
&x_{k+1}^I = \Pi A x_k^I + L\left[y_k - Cx_k^I\right] + B(CB)^+ y_{k+1} +\\
&+ (\Pi B_0 - LD_0)u_k^0 - B(CB)^+ D_0 u_{k+1}^0,\\
&u_k^I = -\left[(CB)^+ CA + B + LC\right]x_k^I + B^+ L y_k + (CB)^+ y_{k+1} -\\
&-\left[(CB)^+ CB_0 + B^+ LD_0\right]u_k^0 - (CB)^+ D_0 u_{k+1}^0.
\end{aligned} \tag{12}
$$

4. Solution of the task

From a comparison of the obtained equations (10), and (12) with the corresponding equations of "classical" inverse dynamic systems [5, 10, 11], it follows that the implementation of the considered method of structural synthesis is equivalent to the introduction into the structure of the inverse system of additional state feedbacks with variable matrix coefficients gain, which makes it possible to change the dynamic properties of the input signal reconstruction error $e_k^u = u_k - u_k^I$, which in turn is described by the equations:

$$
e_{k+1}^x = F(L)e_k^x,\ e_0^k = e_{x_0},\ e_k^u = -\left[(CB)^+ CA + B^+ LC\right] e_k^x. \tag{13}
$$

Let us specify the form of the matrix $F(L)$ (13), that determines the dynamics of the inverse system. Let us represent the matrices of the object (1) in block form

$$
A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ B_2 = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix},\ C = \begin{pmatrix} I_q & 0_{q,n-q} \end{pmatrix},
$$

reduction which is possible with $rank\, C = q$. Then, by the following relations

$$
\begin{gathered}
F = (I_n - L'C)\Pi A - L''C,\\
H = B(CB)^+ + L'\Omega, \qquad (14)\\
G = (I_n - L'C)\Pi AH - L''(I - CL')\Omega,\\
\Pi = I_n - B(CB)^+ C,\ \Omega = I_q - (CB)(CB)^+, \qquad (15)\\
G_0 = (I_n - L'C)\Pi B_0 - GD_0,\ H_0 = -HD_0
\end{gathered}
$$

have

$$
\Pi = \begin{pmatrix} \Omega_{B_1} & 0_{q,n-q} \\ -B_2 B_1^+ & I_{n-q} \end{pmatrix}, \tag{16}
$$

$$
F(L) = \begin{pmatrix} \Omega_{B_1} A_{11} - L_1 & \Omega_{B_1} A_{12} \\ \tilde{A}_{21} - L_2 & \tilde{A}_{22} \end{pmatrix},
$$

where $L', L'' \in R^{n\times q}$ – arbitrary matrices.

Since $C\Pi = \Omega C$, then

$$
F = \Pi A - LC,\ L = L'\Omega A + L'',
$$

and without loss of generality, we can put

$$
L' = 0,\ \tilde{A}_{2i} = A_{2i} - B_2 B_1^+ A_{1i},\ i = 1,2,\ \Omega_{B_1} = I_q - B_1 B_1^+.
$$

The above expressions for determining the current value of an unmeasured input signal contain pseudo-inverse matrices. It is clear that the quality of control processes of the synthesized control system significantly depends on the accuracy of determining the unmeasured input signal by (5). Given this circumstance, there is a need to use effective algorithms for the pseudo-inversion of overdetermined matrices.

5. Using stable algorithms in calculating the pseudo-inverse of the matrix

It is known [8, 12-14] that the problem of calculating a pseudoinverse matrix is generally unstable concerning errors in specifying the original matrix. In this case, the errors in the initial data naturally depend on the accuracy of the experimental studies, and the characteristics of the calculated process depend on the degree of adequacy of the model to the real process. The influence of rounding errors produced during the implementation of the computational procedure on the accuracy of the desired solution can be analyzed based on known analysis methods and a balance of accuracy.

To pseudo-inverse the matrix $K = \begin{pmatrix} B \\ -- \\ D \end{pmatrix}$, we will use the method of minimal pseudo-inverse matrix [15]. This method is based on solving an extremal problem: find a matrix $\tilde{K}_h \in R$, for which

$$\left\|\tilde{K}_h^+\right\| = \inf \left\{\left\|K^+\right\| : K \in R, \qquad \left\|K - K_h\right\| \le h\right\}. \tag{7}$$

In [12] the solvability of the problem (7) is shown for any K_h and h.

To consider the sets of matrices:

$$U_h \equiv \{K \in R : \|K - K_h\| \le h\},\ \Sigma_h \equiv \{K \in R :\ K = U\Sigma V^T,\ \|\Sigma - \Sigma_h\| \le h\},$$

where $U\Sigma V^T$, $\Sigma \equiv diag(\rho_1,...,\rho_m) \in R$ – singular decomposition of matrix *K*. From [12, 14] it follows that

$$\|\Sigma - \Sigma_h\|^2 = \sum_{k=1}^{m} (\rho_k - \rho_k^h)^2 \le \|K - K_h\|^2, \quad \forall K \in R,$$

where ρ_k and ρ_k^h ($k = 1,2...$) – singular values of matrices respectively K and K_h.

Therefore inequality $\|K - K_h\| \le h$ – entails inequality $\|\Sigma - \Sigma_h\| \le h$. This embedding yields the relation [15]:

$$\left\|\hat{K}_h^+\right\|^2 = \left\|V_h \hat{\Sigma}_h^+ U_h^T\right\|^2 = \left\|\hat{\Sigma}_h^+\right\|^2 \le \left\|K^+\right\|^2, \quad \forall K \in R_h.$$

Then it can be shown [12, 14, 15] that the minimal pseudoinverse matrix is defined based on the relation

$$\hat{K}_h^+ = V_h \hat{\Sigma}_h^+ U_h^T,$$

where

$$\hat{\Sigma}_h^+ = diag[\theta(\hat{\rho}_1),...,\theta(\hat{\rho}_m)] \in R^*.$$

It should be noted here [14, 15] that to implement the minimal pseudoinverse matrix method algorithm, there is no need to know the exact singular decomposition of the K_h matrix. It is enough, using well-known numerical methods [7, 8], to find an “almost singular with accuracy $\mu > 0$» decomposition of this matrix [6] and use it in the algorithm together with the new accuracy $h + \mu$.

6. Conclusion

Thus, from the results obtained it follows that the problem of structural synthesis of an inverse system is solvable if the conditions of $rank\begin{pmatrix} \Omega_D CB \\ D \end{pmatrix} = m$ reversibility or observability at the input of the original system are met, which occur only at $q \geq m$, when the number of inputs unmeasured variables does not exceed the number of output ones. In this case, the problem has many solutions, determined up to an arbitrary matrix of varied parameters containing nq elements. This makes it possible to solve at the second stage the problem of parametric synthesis, which consists of selecting the elements of the specified matrix, based on the requirements for ensuring the specified dynamic properties of the inverse system.

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