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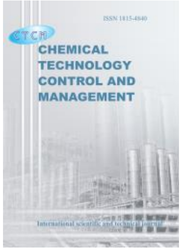

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TOPOLOGICAL INTERPOLATION METHOD FOR MODELING DYNAMIC SYSTEMS

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Abstract. *The method of simulation structural-complex continuous-discrete control systems is discussed. For simulation and calculation of dynamic processes in continuous-discrete systems topological interpolation method is proposed, based on application of hybrid methods of space of state variables and interpolation of signals. The essence of the method is that the dynamics of the investigated system, considered at the final interval, is broken down into subintervals, on each of which the processes are described by linear ordinary system differential equations. The computational efficiency of the proposed method was evaluated by comparison with standard methods such as the Runge-Kutta-Merson method. The use of this method to calculate the dynamic processes described by the non-linear or piecemeal differential equations with the right breaking part allows to reduce the number of calculations by* 2^{n-1} *times compared to the known methods and to eliminate operations related to decomposition of the fundamental matrix in the Taylor power series.*
Keywords. *discrete control system, simulation, nonlinear dynamic systems, dynamic graph models, graph, matrixes.*

Аннотация. *Структуравий-мураккаб узлуксиз-дискрет бошқариш тизимларини моделлаштириш услубияти муҳокама қилинган. Узлуксиз-дискрет тизимларда динамик жараёнларни моделлаштириш ва ҳисоблаш учун ҳолат ўзгарувчиларининг гибрид усулларидан фойдаланиш ва сигналларни интерполяциялашга асосланган топологик интерполяция усуллари таклиф этилган. Усулнинг моҳияти шундан иборат-ки, тадқиқ этилаётган тизимнинг чекланган интервалда кўриб чиқиладиган динамикаси субинтервалларга бўлинади, уларнинг ҳар бирида жараёнлар чизиқли оддий дифференциал тенгламалар билан тавсифланади. Таклиф этилаётган усулнинг ҳисоблаш самарадорлиги Рунге-Кутта-Мерсон усули каби стандарт усуллар билан таққослаш орқали баҳоланади. Тўғри узлуксиз қисми бўлган ночизиқли ёки бўлакли-чизиқли дифференциал тенгламалар билан тавсифланган динамик жараёнларни ҳисоблаш учун ушбу усулдан фойдаланиш маълум усулларга нисбатан ҳисоблаш жараёнини* 2^{n-1} *марта камайтиришга ва асосий матрицанинг Тейлор қаторига ёйиш билан боғлиқ амалларни бажариш имконини беради.*
Таянч сўзлар: *дискрет бошқариш тизими, моделлаштириш, ночизиқли динамик тизимлар, динамик топологик моделлар, графлар, матрицалар.*

Аннотация. *Обсуждается методика моделирования структурно-сложных непрерывно-дискретных систем управления. Для моделирования и расчета динамических процессов в непрерывно-дискретных системах предложен топологический интерполяционный метод, основанный на применении гибридных методов пространства переменных состояния и интерполяции сигналов. Суть метода заключается в том, что динамика исследуемой системы рассматривается на конечном интервале и разбивается на подинтервалы, на каждом из которых процессы описываются линейными обыкновенными дифференциальными уравнениями. Осуществлена оценка вычислительной эффективности предлагаемого метода сравнением со стандартными методами – такими, как метод Рунге-Кутта-Мерсона. Применения данного метода для расчета динамических процессов, описываемых нелинейными или кусочно-линейными дифференциальными уравнениями с правой разрывной частью, позволяет уменьшить количество вычислений в* 2^{n-1} *раз по сравнению с известными методами и избавить от операций, связанных с разложением фундаментальной матрицы в степенной ряд Тейлора.*

Ключевые слова. *дискретная система управления, моделирование, нелинейные динамические системы, динамические топологические модели, граф, матрицы.*

Introduction

A characteristic feature of modern control objects is complexity, multidimensional, nonlinearity, discreteness of control systems, which require special, non-standard control methods to provide the required operational capabilities, and a wide range of functional capabilities for formation of expedient behavior, as well as planning of the sequence of operations with the possibility of forecasting and taking into account the impact of the external environment, as well as active adaptation of its current states [1, 2].

In the case of complex nonlinear dynamic systems, the search for a solution to the problem of controlling the dynamic properties of an object in order to create a control system becomes difficult, that is, it requires large computational costs, is accompanied by cumbersome calculations, and the resulting result is often difficult to analyze and generalize. One of the main issues in simulation is to increase the computational effect, that is, to increase the accuracy and reduce the calculation time, especially for complex nonlinear dynamic objects [3].

The creation of high-efficiency automatic systems faces difficulties related to the complicated structure and parameters of modern controlled objects, which put as priorities the creation of universal machine-oriented methods and models of formalized description, analysis and simulation of complex nonlinear dynamic systems. This paper proposes a method based on representing the system as a space of state parameters to study the development of nonlinear systems [4, 5].

The variety of types and classes of mathematical models describing the dynamics of control systems, the great structural complexity, and the dimension of models make the following approach to process calculation preferable:

- calculation of complex systems on parts;
- taking into account in the dismembered model the forces present in real systems;
- unity of approach to calculation of different types and classes of systems;
- formalizable and easy automation of all simulation stages;
- multilevel representation of processes, both at design and operation stage.

The developed topological interpolation approach of simulation meets the requirements and is intended for simulation structurally complex dynamic systems, including continuous and discrete linear and non-linear structural elements [6].

Algorithms developed on the basis of topological interpolation method allow to automate the process of investigation of dynamics functioning of control system by dynamic object, described by deterministic linear continuous, discrete and nonlinear models.

The topological interpolation approach is based on the use of dynamic graph models, which allow to consider different types of mathematical models from a single position and to determine the connection of system state parameters; vector representations of state parameters, which allow to decompose a structure-complex system and perform calculations in parts; special approach of determining the sequence of calculation of elements of state variables vectors, which ensures high accuracy of simulation complex systems [7].

When using topological interpolation methods, calculations use the vector representation of the input and output variable links, which completely preserves the process property. The topological interpolation approach allows to simulate a system containing various undifferentiated, discontinuous functions in a convenient form. At the same time while keeping peculiarities of the method of using state parameters in the calculation of linear processes, in a generalized form, the connection of structural states (linear sections) of the system is performed [8, 9].

Materials and methods

Modern automatic control theory has a sufficient number of effective methods for calculating linear systems. Therefore, an important problem is the development of a comprehensive theory for discrete dynamic nonlinear automatic control systems. For simulation and research, analysis and synthesis of discrete nonlinear dynamic control systems, a large range of useful methods have been created, such as z -transformation method and others.

However, despite a long and rich history of the issue, traditional methods result in cumbersome, uncomfortable to calculate and design control even for certain types of linear one-dimensional systems.

Modern control systems are characterized by various complexity factors inherent in such systems, such as structural complexity of controlled objects, variability of structures and parameters, and others. Major difficulties of principle arise when all or several of these factors are combined in systems [10, 11].

The complication of systems has updated the problem of developing universal mathematical models, methods of research, calculation and design, covering on a single conceptual basis a theoretical-multiple approach. However, known mathematical methods of decomposition and aggregation have limited applicability because they cannot account for these features of structural-complexity systems.

Development of computer complexes automating simulation processes and analysis of nonlinear dynamic systems in connection with increasing requirements for quality of automatic control systems is a pressing task, which involves modern technologies of mathematical simulation of continuous and discrete processes.

Analysis of simulation methods used in modern computing systems shows that today the following approaches to research of nonlinear discrete systems are used:

- representation of system behavior by the sequence of classical dynamic systems;
- simplification of the continuous part and use of simulation methods.

Thus, today there are no approaches to simulating and analysis of nonlinear dynamic systems, in which methods of research of discrete and continuous components are equally implemented.

Therefore, it seems necessary and natural to introduce in the future the technology of simulating and analysis of nonlinear discrete systems, combining elements of the instrumental base of continuous and discrete mathematical simulation, as well as the technology of symbolic calculations. This new technology should be based on the principle of decomposition of the system into two equivalent components - continuous and discrete, and use of optimal simulation and analysis tools for each of them. Such technology will simplify the procedure of simulation, the principle of automated methods of qualitative analysis of theory of nonlinear systems for discrete component and for discrete periods of behavior of the system [12, 13].

Research results

One way of describing the structure of a system is known to be their graph representation, which is more economical and compact when formalized. The most basic, illustrative of the causal relationship between variables in the system. The combination of the finite set of variables X characterizing the division of the system into parts according to any characteristics, and the binary ratio R on the set X is called system S. As elements it is possible to take input, output values of individual elements of the investigated system [14].

In this case, the structure of the control system under investigation can be described as:

$$S = (X, R_w),$$

where $X = \{x_1, x_2, \ldots, x_n\}$ - multiple variables (vertices); $W = \{w_1, w_2, \ldots, w_n\}$ - multiple weights of arcs (transfer functions).

The weighed binary relation: $R_w \subset \{x \times x\} \times W$, $R_w = \{(x_1, x_2, W_1) \ldots\}$.

At the same time formally the dynamics of the control system is described on a theoretical-multiple approach in the form of graphs as follows:

$$G_t = (X'_t, X''_t, V_t),$$

where

$$X_t = X'_t \cup X''_t; X'_t \cap X''_t = 0;$$
$$\forall x, y \in X_t [(x, y) \in V_t \Rightarrow x \in X'_t \& < y \in X''_t];$$
$$X'_t = (X'_1(jT), X'_2(jT), X_k(jT)),$$
$$X''_t = (X_1(\overline{j+1}T), X_2(\overline{j+1}T), \ldots, X_k(\overline{j+1}T)),$$
$$\forall (x'_i, x''_j) \in V_t [\Diamond (x'_i, x''_j) = \nabla x''_j, x''],$$

where: $\Diamond (x'_i, x''_j) \Leftrightarrow$"arc weight $(.,.)$" $\nabla x'', x'' \Leftrightarrow$"graph tran sfer between nodes$(.,.)$".

Taking into account all the information that a continuous signal carries and using it to calculate processes allows a vector representation of the $\vec{x}_i(t_1)$ variables, which characterizes the free behavior of the system at the i-th point over a certain time interval $[t_1 > t_2]$ when there are no switches or any other modifications in the system breaking the signal, which is equivalent to setting the function $x_i(t)$, where $t_1 \le t \le t_2$.

The accuracy of the calculation is determined by the number of derivatives taken into account in the vector (the more derivatives are the higher accuracy) and the length of the interval $[t_1, t_2]$ with which the inverse dependence is found. If there is a switching or any structural change in the system at the moment t_3 affecting the signal at the i-th point at $t_3 < t_2$, vector $\vec{x}_i$ characterizes the behavior of signal $x_i(t)$ at interval $[t_1, t_3]$.

The calculation of system state parameters must be carried out in accordance with the sequence of propagation of applied perturbations throughout the system. In view of the complexity of the dynamic system structure, the presence of contours, multipolar elements, adders, etc., it is necessary to pay special attention to the properties of different types of dynamic links and the peculiarities of passing signals in them in order to reveal the necessary sequence of calculation of elements of state parameter vectors [15, 16].

The links of a continuous dynamic system can be divided into three main classes, taking into account their dynamic properties: noninertial, inertial, anticipatory. According to this classification, noninertial links such as amplifier, adder, nonlinear functions, that is, at the output of which changes in variables occur simultaneously with changes in input variables. Inertial ones include integrating, aperiodic links, etc., on which when the input variable changes at the same moment, the output variable does not change its value, but its derivatives change. Pre-emptive ones include differentiating links at which the output variable responds instantly to changes in even derived signals.

Ignoring the properties of the signal passing through different types of dynamic links when calculating by parts of processes in a system consisting of a serial chain of dynamic links and a rule for determining the sequence of calculations will lead to errors. For example, if for a chain consisting of n series-connected aperiodic units, the input of which is affected, we will perform calculation of processes with step Δt, sequentially determining the variation of variables in each link while ignoring the instantaneous passage of derivatives in each link while ignoring the instantaneous passage of derivatives of the input variable, then change of output variable of the circuit will be possible only after time $n\Delta t$.

The determination of the values of variables at the next calculated time point t on linear continuous dynamic sections of the system is possible in two ways:

1) the value of the variable break point of the $y(t)$ system is determined by the Taylor formula as a function of the initial state at the same point $\overline{y(0)}$:

$$Y(t) = Y(0) + \sum_{i=1}^{m} \tfrac{Y^{(i)}(0)}{i!} t^i;$$

2) the value of the variable break point at the output of some section of the $y(t)$ system is defined as a function of the initial state of that section of the system and its input vector:

$$Y(t) = f\left(Y(0), \vec{X}(0)\right).$$

Analysis of these paths has shown that the first - more simply, allows after obtaining derivatives to calculate processes, calculations on the second path require preliminary definition of functions of connection f of the output variable with the input vector and initial parameters of the state of the system section. This path is more complicated, but with fewer derivatives calculated, provides higher accuracy of results. Algorithms and programs developed along the second path are faster. Given that communication equations f are formed only once when the system is prepared for calculation, for further use and development, a second way of calculating variables at the sites of a complex dynamic system was chosen [17].

In this case, the dynamics of the system at the local simulation level are represented as a transition state graph, where the transfers of the graph arcs are defined from the expressions:

$$a(t) = e^{-t/T}, a^m(t) = \frac{da^{(m-1)}(t)}{dt^{m-1}} = \left(-\frac{1}{T}\right)^m e^{-t/T},$$

$$h(t) = k\left(1 - e^{-t/T}\right),$$

$$h^{(m)}(t) = \frac{dh^{(m-1)}(t)}{dt^{m-1}} = \frac{-k}{(-T)^m} e^{-t/T},$$

$$h^{(-m)}(t) = \int h^{(-m+1)}(t)dt + Cm = \frac{t^i}{i'} - Th^{(-i+1)}(t),$$

where Cm - is determined from the condition: $h^{(-m)}(t) = 0$.

From the graphical representation of the process, we determine the components of the output signal at time $t = 0$:

$$y^{(1)}(0) = a^{(1)}(0) \cdot y(0) + h^{(1)}(0) \cdot X(0);$$

$$y^{(2)}(0) = a^{(2)}(0) \cdot y(0) + h^{(2)}(0) \cdot X(0) + h^{(1)}(0) \cdot X^{(1)}(0);$$

$$y^{(m)}(0) = a^{(m)}(0) \cdot y(0) + h^{(m)}(0) \cdot X(0) + h^{(m-1)}(0) \cdot X^{(1)}(0) + \ldots + h^{(1)}(0) \cdot X^{(m-1)}(0).$$

The output at time *t* is defined as a function of the input vector at the initial moment:

$$y(t) = a(t) \cdot y(0) + h(t) \cdot X(0) + h^{(-1)}(t) \cdot X'(0) + \ldots + h^{(-m)}(t) \cdot X^{(m)}(0).$$

The obtained analytical expression for calculating processes at different points of state variables enables to calculate processes with relatively large steps over a given time interval.

The topological interpolation method enables to calculate processes in multidimensional multilinked automatic control systems specified as:

$$\dot{x} = A(t)F_x(x) + B(t)F_U(u),$$

$$y = C(t)\psi_x(x) + D(t)\psi_U(u), \quad x(t_0) = x_0$$

where $x \in R^N$ and $u \in R^M$ - a vector of states and control; $y \in R^S$ - vector of output coordinates; A, B, C, D - numerical matrixes $(n \times n)$, $(n \times m)$, $(s \times n)$, $(s \times m)$ sizes.

$F_X(x) = \left(\varphi_1(x^1), \ldots, \varphi_j(x^j), \ldots, \varphi_n(x^n)\right)$ - a vector operator with coordinate functions $\varphi_j(x^j)$ that depends on only one component of x vector variables x^j.

The proposed formula, when solving piece-linear differential equations with function in the right part, allows expanding capabilities and computational efficiency of solving nonlinear differential equations. The features of the topological interpolation approach are the creation of topological properties of *x* variables, which is necessary in the synthesis of a dynamic object control system with different properties.

The essence of the method is as follows. Dynamics of the investigated system considered at interval $[0, t_k]$ are divided into subintervals corresponding to control cycles, at each of which processes are described by linear systems of differential equations:

$$\frac{dx}{dt} = A(t)x, \; t \in [0, T]. \tag{1}$$

It is known that in qualitative studies, fundamental matrices $F(t)$ are used in solving equations (1), and columns of the normalized fundamental matrix can be obtained in different ways, for example by integration with some numerical method or by decomposition into Taylor series n times for the system of differential equations (1) at initial, which are columns of the unit matrix E.

Given that $F(t)$ is a solution to the Cauchy problem for a matrix system of differential equations:

$$\frac{dF(t)}{dt} = A(t)F(t). \tag{2}$$

At initial conditions $F(t_0) = E$ we get formulas for calculation of $F_i(t)$ based on topological interpolation method.

At the same time for each linear section step of calculation h_i exceeding radius of convergence R_i Taylor row in the vicinity of point $t = t_{i-1}$ for solutions of $F^1(t), \ldots, F^n(t)$ included in (2), n - vector tasks of Cauchy is selected.

Then $F^1(t), \ldots, F^n(t)$ on each subinterval $t \in [t_i, t_{i+1}]$ appears to converge the Taylor row:

$$F(t) = \sum_{k=0}^{\infty} \left[\frac{d^k F(t)}{dt^k} \right]_{t=t_i} \frac{(t - t_i)^k}{k!}. \tag{3}$$

Using a similar approach to the elements of the fundamental matrix $F^i(t)$, $i = 0, 1, \ldots$, we obtain a topological formula for calculating the values of variables $x^i(t)$ and the values of their derivatives for moments t_i:

$$\begin{aligned}
\overset{(n)}{x}(t) &= D(\cdot)\dot{x}(t_0) + D(\cdot)\ddot{x}(t_0) + \ldots + D(\cdot)\overset{(n)}{x}(t_0), \\
\overset{(n-1)}{x}(t) &= D(\cdot)\dot{x}(t_0) + D(\cdot)\ddot{x}(t_0) + \ldots + D(\cdot)\overset{(n-1)}{x}(t_0), \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\dddot{x}(t) &= D(3)\dot{x}(t_0) + D(4)\ddot{x}(t_0) + D(5)\dddot{x}(t_0), \\
\ddot{x}(t) &= D(1)\dot{x}(t_0) + D(2)\ddot{x}(t_0)
\end{aligned}$$

where $D(j) = \sum_{i=0}^{K} \frac{a_i (t_{i+1} - t_i)^{i+1}}{(i+1)!}$, $j = 1, 2, \ldots$, a_i - numerical values of a matrix $A(t)$.

For each subinterval $t \in [t_i, t_{i+1}]$, the fundamental matrix (hereinafter referred to as the k-th discret of the fundamental matrix $F_i(t)$) is defined by the formula:

$$F_i(k) = \frac{h_{i+1}^k}{k!} \left[\frac{d^k F_i(t)}{dt^k} \right]_{t=t_i}, \; k = 0, 1, 2, \ldots.$$

Due to decomposition, the $F_i(k)$ fragments uniquely define the fundamental matrix $F_i(t)$ for each linear section as follows:

$$F_i(t) = \sum_{k=0}^{\infty} \frac{(t - t_j)^k}{h_{j+1}} F_j(k), \; t \in [t_j, t_{j+1}], \; j = 0, 1, 2, \ldots. \tag{4}$$

It is obvious that:

$$F_i\left(t_{j+1}\right)=\sum_{k=0}^{\infty}F_j(k)=F_{j+1}(0),\ i=0,1,2,\dots . \tag{5}$$

Moving from the continuous argument t to the area of the discrete argument k, we get a recurrent difference ratio:

$$\frac{k+1}{h_{i+1}}F_i(k+1)=\sum_{l=0}^{k}A_i(l)F_i(k-l), \qquad k=0,1,2,\dots , \tag{6}$$

where $A_i(k)$ - the image of a matrix $A(t)$ at $t\in\left[t_i,t_{i+1}\right]$.

Taking into account (4) and (5), as well as:

$$F_i(0)=\sum_{lk=0}^{\infty}F_{i-1}(k),\ i=1,2,\dots ,$$

$$F_0(0)=E\ . \tag{7}$$

for $i=0$ sequentially get all the discretes:

$$F_i(k+1)=\frac{h_{i+1}}{k+1}\sum_{l=0}^{k}A_i(l)F_i(k-l) \tag{8}$$

From here we will get – at $i=0$

$$\text{for } K=0,\ F_0(1)=h_1\left[A_0(0)F_0(0)\right],$$

$$\text{for } K=1,\ F_0(2)={h_1}/{2}\left[A_0(0)F_0(1)+A_0(1)F_0(0)\right], \tag{9}$$

$$\text{for } K=2,\ F_0(3)={h_1}/{3}\left[A_0(0)F_0(2)+A_0(1)F_0(1)+A_0(2)F_0(0)\right],$$

etc.

According to (7) $F_0(3)=\left[F_0(0)+F_0(1)+F_0(2)+\dots\right]=F_1(0)$.

Similarly at $i=1$ for $K=0$ $F_1(1)=h_2\left[A_1(0)F_1(0)\right]$.

$$\text{for } K=1,\ F_1(2)={h_2}/{2}\left[A_1(0)F_1(1)+A_1(1)F_1(0)\right], \tag{10}$$

$$\text{for } K=2,\ F_1(3)={h_2}/{3}\left[A_1(0)F_1(2)+A_1(1)F_1(1)+A_1(2)F_1(0)\right] \text{ etc.}$$

Formulas (8), (9), and (10) allow a fundamental matrix to be obtained by transferring an unknown initial value from point $t=t_0$ to point $t=t_i$ directly through the values of matrix $A(t)$ and its derivatives, which is convenient for edge tasks in non-uniform differential equations. Derivatives included in (3) can be obtained by sequential differentiation of the right part of the system of matrix differential equations (2):

$$\frac{d^2F_i(t)}{dt^2}=\frac{dA(t)}{dt}\cdot\Phi_i(t)+A(t)\cdot\frac{dF_i(t)}{dt}=\left[\frac{dA(t)}{dt}+A(t)\cdot A(t)\right]\cdot F_i(t)=B_2(t)\cdot F_i(t),$$

where $B_2(t)=\dfrac{dA(t)}{dt}+A(t)\cdot A(t)$.

The mathematical induction method for the K-th derivative of $\dfrac{d^kF(t)}{dt^k}$ gives the expression:

$$\frac{d^kF(t)}{dt^k}=B_k(t)\cdot F_i(t),\ k=0,1,2,\dots ,$$

where $B_k(t)=dB_{k-1}(t)/dt+B_{k-1}(t)\cdot A(t)$, $B_0(t)=E$, $B_1(t)=A(t)$.

Then in the first section of the change of the independent variable $t\in\left[t_0,t_1\right]$, the fundamental matrix is defined by the following expression:

$$F_i(t) = \left[\sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!} \cdot B_k(t_0) \right] \cdot F(t_0) = P_1(t) \cdot F_i(0),$$

where $P_1(t) = \sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!} \cdot B_k(t_0)$. For moment $t = t_1$ $F_i(t_1) = P_1(t_1) \cdot F_i(t_0)$.

And thus it is possible to define for any subinterval $t \in [t_i, t_{i+1}]$ of the linear section an expression for calculating the fundamental matrix in the form:

$$F_j(t) = \sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!} \cdot B_k(t_i) F(t_i) = P_{i+1}(t) \cdot F_j(t_j) = P_{i+1}(t_{i+1}) \cdot P_{i+1}(t_{i-1}) \cdot \ldots \cdot P_2(t_2) \cdot P_1(t_1) \cdot F_j(t_0),$$

$$P_{i+1}(t) = \sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!} \cdot B_k(t_i), \ i = 0,1,2,\ldots,m-1.$$

Formula (6) expresses the discreteness of the fundamental matrix on any of the $t \in [t_i, t_{i+1}]$ subintervals directly through the values of the matrix $A(t)$ and its increase. Then $B_k(t_i)$ is defined as follows:

$$B_0(t) = E;$$

$$B_1(t) = A(t);$$

$$B_2(t) = \frac{dA(t)}{dt} + A^2(t);$$

$$B_3(t) = \frac{d^2A(t)}{dt^2} + 2A(t) \cdot \frac{dA(t)}{dt} + A^2(t) \cdot \frac{dA(t)}{dt} + A^3(t);$$

that is, as Newton binom.

The transition moments from one linear section to another are determined by equating the vector of function $F_i(t)$ and its derivatives composed of matrix $A(t)$ to the value of the nonlinear function $Z(t)$ i.e.:

$$Z_j(t) = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \frac{F_i(t)}{(i+1)!} (t-t_j)^k, \ j = 0,1,2,\ldots.$$

Discussion

The proposed approach reduces the number of calculations by 2^{n-1} and eliminates the operations of $A(t)$ decomposition in the Taylor power series, which is very advantageous with discharged matrices.

The method of calculating processes of complex systems by topological method using interpolation series includes the best sides of topological and interpolation methods. Since this method takes into account the structural states of the system, the rate of change of the process at different points of the system, and the calculation of the process itself is reduced to simple arithmetic operations, replacing the calculation of originals, the finding of which for complex systems is hard work.

The error of the process calculation by the topological interpolation method can be determined using the truncated norms of the derived matrix from the decomposition of the process into a series. The error in the calculation of the processes mainly depends on the selection of the number of members m and the integration step h.

The choice of a step at the set error of calculations σ_k is determined by a formula:

$$h = \frac{1}{m} \sqrt[m+1]{\frac{\sigma_k}{k} (m+1)!}.$$

Approximate initial value of calculation step h_0 - is selected according to relation:

$$h_0 = 0.1 \cdot \min \{T_j\},$$

where T_j - multiple time constants of dynamic elements of the simulated system.

This assumes limited control vector norm, i.e.:

$$\|U_k\| \le \delta .$$

The matrix elements *A* and *C* are then determined from the ratios:

$$H \approx H_1 = \sum_{l=0}^{r} \frac{A^l h^l}{l!} \;,\; F \approx F_1 = \sum_{l=0}^{r} \frac{A^l h^{l+1} B}{(l+1)!} .$$

In this case, the remaining members of the matrix series H and F are equal to:

$$H_2 = \sum_{l=r+1}^{\infty} \frac{A^l h^l}{l!} ,\; F_2 = \sum_{l=r+1}^{\infty} \frac{A^l h^{l+1} B}{(l+1)!} .$$

In this case, the calculation error is determined by the norm

$$\|H\| = \left(\max_{1 \le i \le n} \lambda_H^i\right)^{\frac{1}{2}} ,\; \|F\| = \left(\max_{1 \le i \le n} \lambda_F^i\right)^{\frac{1}{2}} ,$$

where λ_H^i and λ_F^i - own numbers of matrixes $H^T H$ and $F^T F$. It can select the number of members of a series from the specified accuracy.

Based on this decomposition method, we calculate the calculation error for the 5-member limited decomposition series transfer function $x(s)$, while ensuring the solution stability. Then the transfer function of each channel is represented in this form:

$$X(s) = \frac{a^0}{s} + \frac{a^1}{s^2} + \frac{a^2}{s^3} + ... + \frac{a^n}{s^{n+1}} .$$

Using Laplace transform:

$$L^{-1}\left[\frac{1}{s^n}\right] = \frac{(t - t_0)^{n-1}}{(n-1)!} , \tag{11}$$

we find:

$$X(t) = a^0 + \frac{at}{1!} + \frac{(at)^2}{2!} + ... + \frac{(at)^{n-1}}{n!} . \tag{12}$$

When limiting the row to the 5th members of the row $n = 5$:

$$X(t) = a^0 + \frac{at}{1!} + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \frac{(at)^4}{4!} . \tag{13}$$

It is known that the mode of the uniform differential equation is defined by formula (13), where $t = h$ - is the initial condition, then for $t = 0$ we define:

$$X(0) = X(h)\left[1 - \frac{ah}{1!} + \frac{(ah)^2}{2!} - \frac{(ah)^3}{3!} + \frac{(ah)^4}{4!}\right] \tag{14}$$

Subtracting from (12) - (14):

$$X(t) = X(0)\left[1 + \frac{ah}{1} + \frac{(ah)^2}{4} + \frac{(ah)^3}{12} + \frac{(ah)^4}{48}\right] * \left[1 - \frac{ah}{1} + \frac{(ah)^2}{4} - \frac{(ah)^3}{12} + \frac{(ah)^4}{48}\right]^{-1} . \tag{15}$$

For $a = -0.5$, $h = 1$, $x(0) = 1$ define $X(t)$ by formulas (11), (14), (15):

$$e^{-0.5} = 0.606531;$$
$$x(1) = 0.606771;$$
$$x(1) = 0.717757;$$
$$x(1) = 0.606678.$$

Error of calculations (11), (14), (15):

$$\varepsilon = 0.00024 ;$$

$$\varepsilon = 0.111226;$$
$$\varepsilon = 0.000147.$$

The accuracy of approximation for five members of a number of calculated processes is as follows:

$$X(h) = \left[12 - 6ah + h^2a^2\right]^{-1} * \left[12 + 6ah + h^2a^2\right] \cdot X(0). \qquad (16)$$

Calculation error is defined as:

$$\frac{X(h)(15)}{X(h)(16)} = \frac{\left[12 + h^2a^2 + O(h^5)\right]^{-1}}{\left[12 + h^2a^2 + O(h^5)\right]} = 1 + O(h^5).$$

From here it can be seen that the decomposition of the row into 5 members gives an approximation of $X(h)$ with accuracy to $O(h^5)$.

On the basis of the obtained analytical results of the calculated processes, it can be concluded that the error of calculations depends on various indicators $\varepsilon = f(k,h,n)$, где k - the number of members of the decomposition series, h - the integration step, n - order of the system.

The graphs of these dependencies are shown in Figure 1-4. Figure 5 shows the calculation time dependence on the maximum error of the calculated function in the interval $[0,T]$ and the order of the transfer functions.

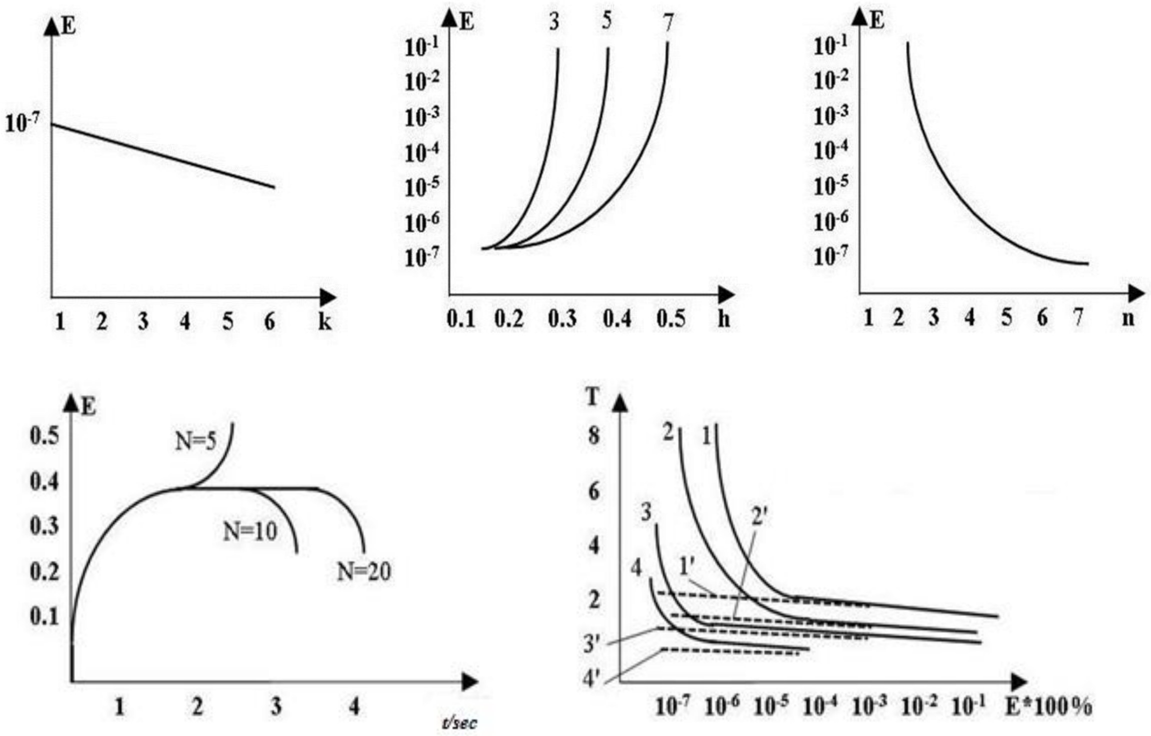

Fig. 1-5. ***Diagrams of dependences of convergence, accuracy and high-speed performance of algorithm (1, 2, 3, 4 – Runge-Kutta method; 1’, 2’, 3’, 4’ - Topological interpolation method).***

Using the topological interpolation method, it is possible to successfully overcome the difficulties of calculating dynamic processes related to the presence of nonlinear links in systems. In this case, the signals in these units undergo nonlinear transformations according to the type of nonlinearities. For continuous type nonlinear links describing their functions, values on the input variable can be differentiated several times.

If the nonlinear link has a relay or piecemeal-continuous character, when the function describing it cannot be diffused across the input variable of the link over the entire range of its determination, it is necessary to predict the moments of transition from one linear section of the nonlinear characteristic to another. In this case, comparisons are made to determine the value and rate of change of the incoming signal. This signal has polynomial form, degree of which is equal to maximum degree of system state derivatives taken into account in calculations.

Conclusions

The paper considers the construction of a topological interpolation method for modeling dynamic systems and the following conclusions can be drawn:

1. An algorithm of automated simulation of dynamics of a continuous-discrete control system based on a topological interpolation approach has been developed, which allows to take into account peculiarities of structural states of the system and obtain high accuracy of simulation.

2. A method for calculating the accuracy of the approximation of calculation function is proposed and justified. This method considers changes in the number of terms in a series, integration step and order of investigated system.

3. The developed topological interpolation modeling approach meets the requirements and is intended for modeling structurally complex dynamic systems, including continuous and discrete linear and nonlinear structural elements. This modeling method is based on the use of graph theory (to formalize structural states) and signal interpolation for calculating processes. The application of the topological interpolation method allows one to successfully overcome the difficulties of calculating dynamic processes associated with the presence of nonlinear links in systems.

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