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SYSTEMATIC ANALYSIS OF METHODS OF CONTROL OF DYNAMIC OBJECTS IN CONDITIONS OF NON-MEASURABLE DISTURBANCES

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Abstract: The article presents a systematic analysis of control methods of dynamic objects under non-measurable disturbances. It is important to study all the factors affecting the control object in order to increase the efficiency of the control system. Due to the immeasurable nature of disturbances affecting control objects, it is shown that they cause negative effects and increase the complexity of control systems in this case. In the control of dynamic objects of this type, the possibilities of control by indirect measurement of disturbances or by estimation of object position and disturbances have been explored. The use cases, shortcomings and achievements of the mentioned approaches are indicated. Robust, invariant and adaptive control systems for disturbance compensation in the control of control objects with unmeasurable disturbances are analyzed. Based on The analysis, is based on the need to develop control systems of dynamic objects with such uncertainties through combined control systems.

Key words: non-measurable disturbances, indirect measurement, object state estimation, combined control systems, disturbance compensation.


Tayanch so‘zlar: o‘lchanmaydigan g‘alayonlar, bilvosita o‘lchash, obyekt holati baholash, kombinatsiyalashgan boshqarish tizimlari, g‘alayonlarini kompensatsiyalash.

Annotation: В статье представлен системный анализ методов управления динамическими объектами в условиях неизмеримых возмущений. При повышении эффективности системы управления важно изучить все факторы, влияющие на объект управления. Ввиду неизмеримости возмущений, оказывающих внешней средой на объекты управления, показано вызываемое ими негативное воздействие, что приводит к повышению сложности систем управления. При управлении динамическими объектами такого типа изучены возможности управления возмущениями путем косвенного измерения или оценки состояния объекта и возмущений. Указаны случаи использования, недостатки и достижения упомянутых подходов. Проанализированы робастные, инвариантные и адаптивные системы компенсации возмущений при управлении подобными объектами с неизмеримыми возмущениями. На основе проведенного анализа обоснована необходимость разработки систем управления существующими динамическими объектами посредством комбинированных систем управления.

Ключевые слова: неизмеримые возмущения, косвенные измерения, оценка состояния объекта, комбинированные системы управления, компенсация возмущений.
1. Introduction:

One of the important goals of the issue and practice of high-efficiency management of technical systems is the assessment of external disturbances affecting the management object and their compensation. It was this issue that led to the development of the theory of automatic control. The problems of accurate alignment of dynamic objects and the problems of compensating disturbances related to it were known already in the last century and continue to this day. Many scientific articles and works have been written about it [1-6].

There are countless factors that can cause chaos in technical systems. Examples of these are negative effects of the environment (non-parametric disturbances), and changes in automatic systems (parametric disturbances).

Uncertainties and shortcomings of mathematical expressions and models in the design and construction of technical systems also cause riots [7]. Methods, tools and directions aimed at compensating all solutions that because riots are of practical importance. Before considering compensation issues, this issue can be divided into two types [8, 9]: critical disturbance compensation and dynamic compensation. If the highest order of the derivative related to the disturbance effect is known (which means the last order to know the Taylor series), by choosing the appropriate order of astatism in the control, the disturbances can be fully compensated when the system is in steady state, as well as partially compensated in transient modes. However, it is known that if the order of astatism increases, the stability of the system becomes a problem. Therefore, compensating for very fast-changing perturbations in this way becomes a problem even in shortistics. Solving the problem of dynamic compensation leads to compensating in the static case as an optional condition. Therefore, we consider the problem of compensation of disturbances as dynamic compensation.

The nature of the issue of compensating riots shows that in the practice of any scientific field of management, there are ways and directions of solving the issue of compensation to one degree or another. In some cases, the mentioned problem is the basis for the emergence of major trends in control theory (invariant control, optimal filtering theory, etc.). In other cases, the direction of compensation of disturbances has acquired an “auxiliary” – “special” tone, that is, it appears in the process of solving other problems (a variational approach to the problem of invariance in the scope of the theory of optimal control). The probable presence of critical routes is determined by a set of different solutions. These include [8, 10]:

- mathematical models used for control objects (linear, non-linear, stationary, continuous, discrete, etc.)
- possibilities of direct measurement of disturbances;
- determination of channels of direct propagation of riots;
- the volume of some a priori data on some descriptions of the riots (type of impact, limits of change, structure and parameters of mathematical models, etc.).

Since the existing directions for solving the problems of disturbance compensation are complex and diverse, it is appropriate to combine them into the following scientific directions: optimal, robust (regular), invariant and adaptive control [11-21]. Active compensation involves adding additional effects to the system to compensate for disturbances. Slow compensation means creating such a structure in the object of control, in which the quality of control does not deteriorate even if disturbances change within certain limits [19].

Active compensation methods include invariance theory methods and adaptive control methods [9, 16]. Briefly, invariance theory methods allow full perturbation compensation in many cases, but the data must be very complete. In adaptive control methods [17, 22], on the contrary, it is not required to have a priori information about the disturbance. All the necessary information is formed during the operation of the adaptation algorithm.

Weak compensation methods include robust control [19-22] and optimal control [11-14]. In robust control methods, disturbances are focused on ensuring the quality of control at a predetermined level when \( V(\hat{e}) \leq 1, (\hat{e} = 1, 2, ..., n) \) changes arbitrarily within a certain limit.
Optimal control methods apply to control objects that are random disturbances. In them, the problem of speeding up the control law is set in such a way that the given goals of control are satisfied, and minimization of the generalization of the classical type stochastic functional or functional is ensured. At the same time, it also minimizes the impact of riots that try to derail the object.

In the theory of random processes [23-25], the optimal control system (in the sense that the mean quadratic deviation is minimal) consists of the state vector \( x(t) \) and its estimate \( \hat{x}(t) \). This estimate builds for an optimal control system and requires accurate measurement of condition. This feature is called the separation principle and is used to speed up the optimal control system. The optimal position of the object is estimated by the minimum of the mean square deviation. This approach to the problem of automatic control system (ACS) accuracy is based on probabilistic characteristics of disturbances.

In optimal control methods, the problem of compensating disturbances is brought to the problem of optimal evaluation, therefore, the effectiveness of compensation is determined by the effectiveness of evaluation, and all conditions are assumed to be the same. Therefore, only the methods of evaluating signals in the conditions of disturbances that clearly affect the object will be considered.

Thus, one way to approach the compensation problem is to estimate either the perturbations themselves, or the sum of the associated signals (for example, variable parameters of the object’s state). The next situation is called the indirect measurement of disturbances.

2. Methods of evaluation of non-measurable disturbances

There are many approaches and methods dedicated to non-measurable perturbation estimation [9, 13, 19, 26-28]. All of them can be divided into two groups depending on the level of a priori information about riots [28]:

1) traditional methods of indirect measurement using “differential connections” (“plugs”).
2) evaluation methods based on differential equations of disturbances [29, 30].

A differential circuit requires at least two measurable signals to be present. One depends in some way on an unmeasurable signal. For example, if the input-output equation of a control object is:

\[
A(p)y = B_0(p)u + B_1(p)f,
\]

where \( f \) is the output coordinate and the control is seen as the measured signal to estimate the disturbances. Then, the equation of the device for monitoring the effects of turbulence, implemented on the basis of a differential scheme, will be as follows

\[
H(p)\hat{f} = M(p)y + N(p)u,
\]

where \( M(p) \), \( N(p) \), \( H(p) \) – are operators and must satisfy the evaluation stability of \( \hat{f} \) being formed, the realization of the equation and the given quality (convergence rate).

From the analysis given in [28], it is known that the synthesis of the device for monitoring the effects satisfying all the above conditions is possible only for objects without inertia. Otherwise, the synthesis of the device for monitoring the effects is possible based on the two-channel principle, even then only if the \( B_1(p) \) polynomial in (1) is equal to the order of the object and has a non-minimizable constituent. In this case, the influence of the \( \hat{f} \) value on \( f \) is determined by the zeros of the polynomial, and to change it, only the property of the object needs to be changed [28]. If the conditions are more complex than stated, of the device for monitoring the effects can be constructed for only a few unmeasured effects. In this case, some equivalent effect given to the object’s scalar “input” is also evaluated. However, the conditions of such citation and the possibilities of viewing the evaluative structures, especially for the multidimensional case, have not been sufficiently studied, compared to the literature.

The second group of methods can include methods that use mathematical models of effects expressed in the system of ordinary differential equations that are brought to the Cauchy form [28-30]:

\[
\hat{f} = Pf, \quad f(t_0) = f_0,
\]
when \( P \) – some square matrix known in advance, \( f_0 \) – unknown vector of initial conditions. Suppose that the control object is represented by the following equations:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Ff; \\
y &= Cx + Df.
\end{align*}
\]

In this case, the synthesis of the evaluation device is performed based on the asymptotic evaluation of the variables of the extended object state \([13, 26, 31]\). The mathematical model of this object combines the initial model of the object and the following perturbation model:

\[
\begin{bmatrix}
\dot{x} \\
f
\end{bmatrix} = \begin{bmatrix} A & F \\ 0 & P \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u;
\]

\[
y = \begin{bmatrix} C & D \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix}.
\]

In this case, the extended object state estimation conditions are connected with monitoring processes, the equations of synthesis of the device for monitoring the effects are performed on the basis of the state monitoring equations of Kalman and Lünberger \([9, 13, 23, 29]\). This approach to perturbation estimation is also used for linear discrete systems \([9, 31, 32]\).

The analysis of the mentioned approach \([28]\) shows that the methods of the second group are also implemented based on the differential scheme, only V.S. Kulebakina's invariance criterion is used (the system in (2) is viewed as the \( K(D) \) – representation of perturbations). In this case, the observation condition (condition) of the expanded object is seen as a necessary condition for the formation of disturbance estimates, and the spectra of the object, the observer, and the impact assessment are filled with the condition that they do not intersect with each other.

It should be noted that the observed approaches to the assessment of non-measurable disturbances are very limited and have many shortcomings in practical application.

For example, the first approach (on the turbulent channel of objects) is limited to a class of linear stationary one-dimensional minimal spatial equations. Perturbations in the object directly affect both the parameters and the "output" of the state variables, and the image of the transfer function has several poles along the perturbation channel, which lie to the left of the spectrum of possible perturbations. It is very difficult to satisfy such conditions in practice.

The effectiveness of the second group of approaches is often limited by the uncertain nature of mathematical models of active turbulence. This character cannot always be represented by linear differential equations. Therefore, the application scheme of the second approaches is limited to constant (constant) or slowly varying disturbances \([13, 33]\).

### 3. Assessment of the condition of objects under the conditions of random effects

Let the equations of an object be as follows:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t), t) + \xi(t); \\
y(t) &= h(x(t), u(t), t) + \eta(t),
\end{align*}
\]

when \( f(\cdot) \), \( h(\cdot) \) – a vector of a nonlinear function in general, \( \xi(t) \), \( \eta(t) \) – Gaussian “white noise” represents uncorrelated external perturbations and measurement errors and is composed of correlation matrices as

\[
E[\xi(t)\xi^T(\tau)] = Q(t)\delta(t - \tau), \quad E[\eta(t)\eta^T(\tau)] = R(t)\delta(t - \tau),
\]

when \( \delta(t - \tau) \) – Dirac function.

There are many definitions of facility condition assessment. The relevant methods and approaches can be classified on the basis of various characteristics, such as the appearance of the mathematical model of the object, the characteristic of observation, the time of observation, a priori information about the influence of noise, etc. Many methods have been developed for fully evaluating
the state of the observed object when statistical descriptions of objects are known [13, 31, 34]. In this case, the following function is minimized:

$$J = \int_{t_0}^{T} (y(\tau) - h(x,u,\tau))^T \cdot R^{-1}(t) \cdot (y(\tau) - h(x,u,\tau)) d\tau,$$

the function is brought to the following vector differential equation related to the corresponding mean square deviation of the estimates in relation to the quantities being evaluated, the equation reflects the optimal observer of the state [35].

$$\dot{x} = f(\cdot) + (x\overline{h}^T(\cdot) - \overline{x}\overline{h}^T(\cdot))R^{-1}(t)(y - \overline{h}(\cdot)),$$

when $\overline{\beta}$ is the conditional mathematical expectation of magnitude $\beta$.

Expression (5) cannot be solved even numerically, which is due to the fact that the Stratonovich equation cannot be solved. However, it is possible to develop various suboptimal algorithms of various evaluations based on it. For this, it is necessary to expand the nonlinear functions $f(\cdot)$ and $h(\cdot)$ into a Taylor series around the resulting or some uninduced motion, as well as other forms of approximation [31] (several terms of the expansion are preserved). Depending on the number of terms to be preserved, suboptimal algorithms of the second and first order are constructed. In practice, first-order algorithms are preferred. These are not even second-order in terms of accuracy of calculations and are economical from the point of view of practical calculations [31]. Among the first-order algorithms, the most famous are algorithms called Kalman-Buys filters (KBFs). When the model of the object is nonlinear (3), the KBF will have the following form:

$$\dot{x} = f(\overline{x},u,t) + P(t) \left( \frac{\partial h(\overline{x},u,t)}{\partial \overline{x}} \right)^T R(t)^{-1}(y - h(\overline{x},u,t)), \overline{x}(t_0) = \overline{x}_0,$$

where $P(t)$ matrix is derived from the following covariance equation:

$$\dot{P}(t) = \frac{\partial f(\cdot)}{\partial \overline{x}} P(t) + P(t) \left( \frac{\partial f(\cdot)}{\partial \overline{x}} \right)^T - P(t) \left( \frac{\partial h(\cdot)}{\partial \overline{x}} \right)^T R(t)^{-1} \frac{\partial h(\cdot)}{\partial \overline{x}} P(t) + Q(t).$$

The main advantage of the indicated algorithms is the presence of negative feedback in the evaluation circuit. It automatically corrects (corrects) the grade when the connection is made. However, these estimates are only suboptimal if the object is nonlinear, the reasons for this are: equations (6), (7) are solved approximately, defects in linearization, and errors in the selection of initial conditions $P_0$, $x_0$. It is because of these assumptions that the process converges to $P(t)$ and the estimation errors increase. It should be noted that the goal of filtering (when criterion (4) is minimized) does not mean that the estimation algorithms fulfill the stationarity property, therefore, the optimization algorithms in the sense of the mean quadratic deviation may be unstable, which means that they may be invalid in practice.

A linearized model of an object

$$\begin{align*}
\dot{x}(t) &= A(t)x(t) + B(t)u(t) + \zeta(t); \\
y(t) &= H(t)x(t) + \eta(t),
\end{align*}$$

for the KBF equations are also linear:

$$\dot{x} = A(t)x(t) + B(t)u(t) + P(t)H(t)^T R(t)^{-1}(y - H(t)x(t)),$$

$$\dot{P}(t) = A(t)P(t) + P(t)A(t)^T - P(t)H(t)^T R(t)^{-1} H(t) P(t) + Q(t).$$

In this case, the generated estimates are strictly optimal, and the estimation algorithm is called stable [31, 35]. However, Riccati’s matrix equation (8) can be solved only numerically in most cases. Therefore, the values obtained here should be called quasi-optimal. The structure of linear continuous KBF is shown in fig 1.

The stability character of the estimation obtained by means of linear KBF was previously shown that it can be used for non-linear objects as well. The assessment process is divided into cycles. At the beginning of each cycle, the nonlinear model is linearized and a linear estimation algorithm is created.
At the end of the cycle, the value of the state vector is output. The (estimate) model acts as a reference point for linearization in the next cycle. The advantage of cycle evaluation is that it allows the use of a simplified model of the process being evaluated throughout the cycle. When implementing the algorithm numerically, the disadvantages of accumulated errors and numerical drift can be avoided by limiting the evaluation time per cycle. A major challenge in this approach is determining the length of each evaluation interval. This serves to ensure the accuracy of the resulting estimates and the necessary computational efficiency. The choice of the length of the evaluation interval is individual for each specific case, so there are no general recommendations for its effective solution.

Figure 1. Structural diagram of linear continuous KBF.

Achieving the required computational efficiency by eliminating the covariance equation or attempting to replace it with a simpler one leads to the analysis of a number of heuristic algorithms in which some cases are replaced by the ensemble average to the time average. Such algorithms are computationally simpler and more convenient. However, the convergence conditions and operability of these algorithms have not been sufficiently studied [31].

Recurrent algorithms are used if there is a problem of estimating the state of the object in a discrete time interval. In this case, as in continuous cases, the use of linear discrete KBF is more efficient and it is convenient to use EHM [29, 31].

Difficulties in using KBFs and other algorithms for optimal turbulence estimation in realistic settings are due to the uncertainty of statistical recommendations for turbulence. Such descriptions change with the modes of the control object. In this situation, optimal filtering does not justify itself [23] and the tracking problem can be computationally simplified by using an average-tuned tracker or by fitting the tracker to a suitable location (Kalman tracker). There are opinions that a good method of optimal filtering is radar tracking [36] because when the weather conditions do not change, the parameters of the turbulence can be assumed to be known and stationary.

The problem of estimating the state of the object can be considered as the problem of estimation under conditions of uncertainty when the statistical recommendations of noise are unknown [13, 37]. This issue is also applied to objects whose structure is unclear (abstract) later. These are called adaptive control [17-19, 22].

Thus, the optimal control in the conditions of random effects is brought to the problem of optimal estimation of the state of the object. Therefore, the defined control laws are aimed at compensating disturbances, and their order depends on the conditions of the functional compensation form being optimized. Approximate solutions to the optimal control problem combined with quasi-optimal state estimates lead to suboptimal control. If the control issue is synthesized by other methods, the effectiveness of random disturbance compensation will depend on the effectiveness of the applied approach.

4. Compensation for non-measurable disturbances in robust control systems

The robustness of the control system (close to the meaning of regularity) means the roughness of the system in response to slight deviations of the system parameters from the nominal value. The robust approach, unlike the adaptive approach, does not envisage the gradual convergence of the system’s...
performance due to active intervention in management, but it will be possible to maintain the previously intended quality if the negative factors affecting the system are within certain limits. In this sense, such a group of methods can be considered as a weak method that does not evaluate the compensation of disturbances.

The first step in establishing the roughness of the systems started with the issue of preserving the asymptotic stability properties when the parameters of the object change on a small scale. This direction developed sharply after the article of V. L. Kharitonov [31, 38, 39].

The modern stage of the theory of robust control is characterized by the study of not only the stability of the parametric turbulent system but also the preservation of the given control quality [40-44]. For example, in [40, 41] it is recommended to evaluate the quality and stability of the control system by the algebraic-frequency method. The conditions of stability and quality of linear, continuous, robust systems are created based on the indicators included in this work. In [40], modal estimates of robustness were investigated using specially constructed functional, modal, and parametric sensitivities and conditioned numbers of eigenmatrices. These criteria of sensitivity have studied the possibility of choosing the nominal parameters of the control system. In this case, the system will have a nominal sensitivity and various modifications of the modal-robust controller will be synthesized.

Robust systems also include high-gain systems and derivative control systems. However, in such systems, the coarseness property is defined close to the infeasible descriptions of the control problems. It should be noted that the development of the robust control theory of Kharitonov’s theorem is its use in the construction of linear objects and systems.

However, efforts to build coarse-grained control systems for nonlinear objects are rare. The effectiveness of such systems is limited to the nonlinear functional class.

Robust control theory is also divided into linear and nonlinear discrete control systems [43, 44]. For example, nonlinear discrete systems were considered in [44]. They have a linear part and a non-inertial non-linear element and belong to some finite set. In [44], frequency criteria of robust stability for such systems are connected. The issue of robust control is poorly studied in the discrete domain, and its effective solution is limited to low-order objects. Such a situation is typical of continuous control systems.

From the analysis of robust control methods, it can be seen that the considered approach should be used to compensate for parametric disturbances, in particular, disturbances caused by the mathematical model of the invariant part of the system not being accurately represented.

When it comes to compensating external (non-parametric) disturbances, it can be said that the problem of gathering disturbances can be considered as a robust approach [20-22, 43, 44].

If only the modulus of the perturbations acting on the system is known:

$$|v_i(t)| \leq l \quad (i = 1, 2, \ldots, n),$$

if their time variation is not known, studying the performance of a dynamical system under these perturbations becomes a matter of perturbation accumulation. This issue examines how disturbances can derail the system. After studying such a connection between disturbances and the system, it is possible to predict what the maximum deviation of the system will be when a certain constraint is $l$.

The most studied field of turbulence accumulation is linear systems [13]. The operation of the control system is represented by the following mathematical model:

$$\begin{align*}
x(t) &= A(t)x(t) + B(t)z(t) + F(t)v(t); \\
y(t) &= C(t)x(t),
\end{align*}$$

(9)

scalar variables in this: $y(t)$, $z(t)$, $v(t)$ — system output, assignment, and disturbance, respectively.

The forced movement of the system (9) is related to the effect of $v(t)$ and is expressed by the integral of the sum:

$$y_v(t) = \int_0^t w(t, \tau)v(\tau)d\tau,$$

(10)
when \( w(t, \tau) \) – is the weight function of the system’s state of disturbance.

In order for the dynamic error in the system to reach the maximum \( y(t) \) the absolute value of the external disturbance must remain at a maximum, and its sign must change so that the sub-integral function in (10) must maintain its sign in the time interval \([t_0, t]\)

\[
|v(t)| = l, \quad \text{sign}(v(t)) = \text{sign}(w(t, \tau)), \quad t_0 \leq \tau \leq t.
\]

Thus, knowing the weight function \( w(t, \tau) \) and the constraint \( l \), it is possible to determine how much the state of the system deviates. The problem of turbulence accumulation has been studied for nonlinear dynamic systems (static nonlinear) [26, 28, 35]. However, in practice, the models of most objects and processes are represented by analytical nonlinearities (in other typical cases, such a problem cannot be solved for objects and processes).

It should be noted that the above problem is more applicable to the analysis than to the synthesis of dynamic systems. Some elements of the synthesis are seen in [13, 26, 27], and the problem is posed as follows: how to choose the coefficients of the characteristic polynomial, which is a free term, in order to reduce the maximum deviation of the system as much as possible? According to the conclusion made by the authors of the above-mentioned scientific works, the possible deviation of the system is equal to \( 1/a_0 \) (when the fraction has a real root) and greater (if it has a complex root). From this conclusion, it is clear that the relation, \( 1/a_0 \) indicates the increasing property of the system both on the disturbing channel and on the assigned channel. Therefore, changing the coefficient aimed at reducing the dynamic error changes the characteristics of the system according to the task channel. This is contrary to the objectives of management. The disadvantage of this approach is that it is difficult to achieve a dynamic maximum estimate of the error in the case of vector perturbations.

Such a feature of the problem of collecting riots shows that the effectiveness of this direction is not enough.

5. The principle of invariance and the problem of compensation of non-measurable disturbances

According to the proposal of the authors who proposed an approach to control as if it were being studied, the problem of invariant control is posed as follows [15-17, 18, 46]. If the model of the automatic control system is expressed by the following differential equations

\[
\begin{align*}
    a_{11}(D)x_1 + \cdots + a_{1n}(D)x_n &= F_1(t); \\
    &\vdots \\
    a_{nj}(D)x_1 + \cdots + a_{nn}(D)x_n &= F_n(t),
\end{align*}
\]

(11)

when \( a_{ij}(D) \) – second order operator, \( D \) – symbol of differentiation, \( F_i(t) \) – some effect (action), then In order to ensure the absolute invariance (selective invariance) of the \( i \)-coordinate with respect to the \( j \)-perturbation, the minor corresponding to the \( a_{ij}(D) \) element of the matrix system (11) must be zero by definition [38, 42]:

\[
A_{ji} = \begin{vmatrix}
    a_{11} & \cdots & a_{1i-1} & a_{1i+1} & \cdots & a_{1n} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    a_{j-11} & \cdots & a_{j-i-1} & a_{j-i+1} & \cdots & a_{j-1n} \\
    a_{ji+1} & \cdots & a_{j+i+1} & a_{j+i-1} & \cdots & a_{j+i} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    a_{nj+1} & \cdots & a_{n+i-1} & a_{n+i} & \cdots & a_{nn}
\end{vmatrix} = 0. \quad (12)
\]

A necessary condition for ensuring invariance when performing (12) is that the determinant of the matrix of the following open system must not be equal to zero

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The variational approach to the problem of invariance is applied to both the operators and the concept of invariance in turbulence. Sometimes the concepts of invariance and the chain principle conflict with each other. Relying on the turbulence output channel, the absolute invariance of the operator acts as an information channel. The absolute invariance of the operator is sought, the disturbances must be distributed between two channels. This problem is put in a variational form.

The absolute invariance of the \( x_i \) coordinate with respect to the \( F_j(t) \) perturbation is essentially zero for the corresponding channel transfer functions. A sufficient condition for the implementation of absolute invariance is the physical possibility of compensating chains to form an additional channel.

A different approach to building invariant systems. Suggested in the scientific works of V.S. Kulebakin - \( K(D) \) – is an image [19, 28, 45]. The absolute invariance of the operator acts as an information channel.

\[
K(D) : K(D) * F(t) = 0,
\]

in which the structure and parameters of the operator are unknown.

In this case, it is necessary to choose the parameters of the compensating circuit in the synthesis problem in such a way that the final transfer function on the turbulence output channel includes the \( K(D) \) operator as a multiplier. The condition for using this approach is that the spectra of the – \( K(D) \) image and the channel transfer function do not intersect [19, 28].

In this case, the differentials in the form of \( F(t) \) perturbation (13) are accurately represented by the solution of a linear, stationary, homogeneous equation, \( F(t) \) is absolutely invariant with respect to the system. Otherwise, invariance is observed.

The problem of invariance is also put in a variational form. For the system expressed by differential equations of the Cauchy form, it is necessary to find the following conditions for \( f_i \) so that

\[
\frac{dx_i}{dt} = f_i(x_1, \ldots, x_n, F(t), t), \quad x_i(t_0) = x_{i0}, \quad i = 1 \ldots n,
\]

where is the value of the objective function

\[
J = \varphi(x_1, \ldots, x_n, t) \rightarrow \min
\]

and it does not depend on external influence [11, 24]. This problem is fully solved in the case of linear systems. Condition (14) in the optimal sense for these systems is also invariant in the above sense. In this case, the optimal control is formed taking into account the turbulence \( F(t) \) and it should also be possible to measure the \( F(t) \). The variational approach to the problem of invariance is applied to both nonlinear automatic systems and systems with variable coefficients [9].

It should be noted that, in general, the use of control methods in conditions of invariance is limited by a number of serious factors. For example, the actual implementation of the compensating chain is rare in practice. One of the negative factors is that sometimes the concepts of invariance and roughness conflict with each other.
The invariance of disturbances, the impossibility of exact compensation laws, and the complexity of compensation in multidimensional cases all require the use of approximate methods that solve the problem.

The fact that most disturbances cannot be measured limits the possibilities of invariant control. In this case, it is possible to talk about partial compensation, which cannot be measured directly. This work is carried out through estimates based on the results of measurements of measurable variables of the system. Then, in determining the quality of compensation, all other conditions are equal, as the obtained estimates, the accuracy of disturbances becomes decisive in the theory of invariant control.

6. Compensation for non-measurable disturbances in adaptive control systems

If there is no information about disturbances affecting the object or system, such objects are controlled by adaptive control methods [9, 14, 16-19].

In relation to the issue of compensation, all adaptive control issues can be divided into two groups. The first includes the invariance model of the control object and process evaluation algorithms to the adaptive algorithms. The latter includes non-identifiable algorithms of adaptive control.

Adaptive systems with identifiers. The structure of adaptive automatic control systems is largely determined by the principle of division. According to this principle, an optimal (in linear problems) or suboptimal (in nonlinear problems) system consists of an optimal (suboptimal) evaluation and identification and an optimal control system, which is built according to the conditions of exact measurement of the state and vector of system parameters, but uses the magnitudes of these estimates. If parametric or non-parametric identification operations are performed in the system, it is called “identified”. The general structure of an adaptive control system with an identifier is illustrated in Figure 2 below.

Depending on the algorithmic support of subsystems, it is possible to distinguish a large number of variants of adaptive systems of such classes: systems with a phenomenological model, systems with a generalized KBF, systems with a predictive algorithm, etc.

![Figure 2. The general structure of an adaptive control system with an identifier.](image)

The presence of an identifier in the adaptive system (it can be considered as a sensor of disturbances in the parameters) allows using the principle of adjustment for disturbances in the adaptation circuit. In this case, the adaptation contour acts as a second channel for the passage of
disturbances [30, 33, 48]. A current evaluator of an object’s parameters can be constructed based on identifiers that operate on the principles of identifier searching and non-searching. However, in order to effectively compensate for parametric perturbations, it is important that the estimates of object parameters obtained using the identifier are reasonable and efficient in the sense that they approximate the true values of those parameters. As for non-evaluable external disturbances, it can be said that their compensation in the specified class is carried out according to the control law of the circuit.

A disadvantage of this group of methods is the long duration of the identification and evaluation stages. Therefore, identity systems are found to be suitable for stopping slow-changing parametric disturbances. In addition, when the system has many nonparametric disturbances, the identification algorithms lose their value. Therefore, the laws synthesized in this case may violate the quality of management.

Non-identifiable adaptive systems. A group of such systems is designed from the beginning to manage control objects not only in parametric conditions, but also in conditions of structural abstraction. In this regard, the issue of compensating for unmeasurable disturbances, depending on the stated goals, is generally expressed precisely in the control systems of this group.

A common example of the specified group is adaptive systems with reference model (ASRM) [1, 7, 18] (Fig. 3).

![Figure 3. The structural scheme of the standard model adaptive automatic control system: A – adjuster, AB – adaptation block, BM – benchmark model.](image)

The task of ASRM is to ensure that the response (reaction) to the input signal is as close as possible to the response to that signal from the reference model. In this case, the goals of adaptation and management overlap. The advantage of ASRM is the ability to quickly adapt when given input signals of a certain form [1]. In other cases, the adaptation is slow because it is necessary to wait until the transition process is completed to find the estimator [18]. This situation makes it difficult to use ASRM on non-minimal spatial objects. Such systems cannot adapt to external conditions if the input signal being measured remains unchanged [33]. In such systems, it is also difficult to use a command device given in the form of numbers, because the estimator function cannot be given in a clear form, and search algorithms have to be used to achieve the goal of adaptation [2]. The field of rational use of ASRM is limited to linear objects whose transitions are relatively slow compared to changes in variable parameters and external disturbances [49].

The next big group is the methods of management by derivatives. This group includes the methods of higher derivative equations and differential operator equations that were previously discussed. A.S. Vostrikov’s localization method consists of synthesis according to any equations.

It looks like this

\[
\begin{align*}
\dot{x} &= f(t, x) + b(t, x)u, & x \in \mathbb{R}^n, \\
y &= h(t, x), & (u, y) \in \mathbb{R}^l
\end{align*}
\]

for a one-dimensional control object, the admissible differential equation with respect to the highest derivative is given:

\[y^{(l)}_{xx} = F(Y, r),\]
when \( y = [y \ y' \ ... \ y^{(l-1)}] \) – vector of derivatives of the output coordinate.

To determine the \( l \) - output coordinate to determine the layout – \( y(t) \) the influence of management on the derivatives of is analyzed.

The first derivative looks like this:

\[
\dot{y} = f_1(t, x) + Hb(t, x)u, \quad f_1(t, x) = \frac{\partial h}{\partial t} + Hf,
\]

when \( H = [\partial h / \partial x_1 \ ... \partial h / \partial x_n] \).

If \( Hb = 0 \) is clearly connected to by continuing the differentiation \( l \) - the order derivative is found, which is \( H/b \neq 0 \) corresponds to the multiplier.

The governing law is given by the following expression:

\[
u = K [F(Y, r) - y^{(l)}],
\]

when \( K \) – is the amplification factor, and is the derivative of the, \( y^{(l)} - l \) order output signal.

A necessary requirement for the remaining \( (n-l) \) state variables is their stationary behavior. Fulfillment of this requirement is a feature of the object and does not depend on the designer. The selection of the \( K \) coefficient is carried out with the requirements set for the accuracy of the error of the derivative. As the error increases, the control law tends to the following expression:

\[
\lim_{K \to \infty} u = b^{-1}(F - f).
\]

This limit expression control law corresponds to the invariant control law derived by the method of higher derivative equations, but here \( b(t, x) \) it is not required to make the function visible. Instead, the \( l \)- order the highest derivative should be used. The advantage of the method is the presence of perturbations (including non-measurable) of the constructed system and its invariance to the nonlinear nature of the object. Hence the name of the method. This feature allows the method to be applied to non-linear non-stationary objects whose mathematical model is not clear. This can be explained by the inclusion in the control law of the output derivatives, not the right-hand side of the object equations. They are measured using a \( (l+1) \) order low-inertia dynamic loop filter. However, the methodology is still far from perfect: some assertions and assumptions based on it have not been proven, stability in the selection of amplification coefficients and inertiality of filters has not been studied (not researched), and some of the remaining unknown state variables have not been observed. The method does not allow the synthesis of astatic systems.

In [27, 33, 35] an adaptive optimal adjuster with a variable polynomial filter with A.A. Krasovsky structure is proposed. In the identification device, a set of KBFs with different configurations is used in parallel. A linear, simplified model is used as a mathematical model of the controlled object

\[
\dot{x}(t) = Ax(t) + Bu(t), \quad A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \vdots \\ 0 \end{bmatrix}, \quad x(t_0) = x_0.
\]

This equation does not have a single unknown parameter, but maintains its adequacy over short time intervals, so the filters work in cycles. At the beginning of each cycle, the initial conditions of all KBFs are adjusted, and at the end of the cycle, their values are used in the control calculation. The order of the model is evaluated by the order of the KBF in each observation cycle. Then the output signal can be predicted with minimum mean square error. To reduce the amount of necessary calculations, the elements of the covariance matrices are calculated for each KBF and stored in the memory of the identification device. In [33], an adaptive observer is used for the control and identification of fast processes and in critical principles, the control is calculated from the minimum of the generalized
performance function. In [41], this observer works in a modal control system. The advantage of the considered approach is that the control can be built even in conditions where a priori information about the object is minimal.

As shown in [41], when the duration of the evaluation tends to zero, the system is robust to objects expressed as follows, i.e., to one-dimensional controlled objects

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t); \\
\dot{x}_2(t) &= x_3(t); \\
&\vdots \\
\dot{x}_{n-1}(t) &= x_n(t); \\
\dot{x}_n(t) &= \sum_{i=1}^{n} a_i x_i(t) + bu(t) + v(t),
\end{align*}
\]

and is invariant with respect to scalar perturbation.

The main problem of this direction (which also applies to other methods of control by derivatives) is to quickly evaluate the time derivatives of the observer with a noisy signal. In addition, determining the order of the model based on the minimum extrapolation error of the observed signal leads to the fact that it differs significantly from the order of the real object.

The indicated shortcoming is partially eliminated in [49], the proposed method provides an original algorithm for cyclic monitoring of derivatives and results close to the real object in the sense of determining the order of the model.

As for compensation, it should be said that when the disturbance is a vector \( v(t) \) (parametric and nonparametric), it is not possible to achieve full compensation of \( v(t) \) with the built-in approach, even if the object is one-dimensional and the evaluation interval is infinitesimally small [50].

Non-identifiable algorithms of adaptive control are implemented both in self-adjusting search systems [33] in intelligent control systems [51, 52] and in systems built on the basis of the method of recurrent objective inequalities [1, 33, 49]. Without going into detail about these systems, it can be noted that they all have the same shortcoming: adaptation or learning and extreme complexity of algorithmic procedures, these features narrow the field of effective use of the systems.

7. Conclusion

Thus, despite the large number of adaptive control methods, there are few practical achievements in this field [1, 2, 27-29, 53]. However, the great interest in this problem indicates its relevance and the need for further development. It is necessary to simplify the algorithms and expand their capabilities. For example, the use of adaptive control principles in the field of compensation of disturbances. This work differs from the methods of classical control theory by its discrete-continuous nature and gives new results when the effectiveness of continuous compensation methods is insufficient.

References:

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